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AMATEUR CINEMATOGRAPHY AND THE KODACOLOR PROCESS.*

BY

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In August, 1923, a paper describing a new process of amateur cinematography which had just been introduced was published in the JOURNAL OF THE FRANKLIN INSTITUTE. The process depended upon the use of a narrow film, 16 mm. wide, instead of the 35 mm. of the standard motion picture film. This film, after exposure, is not developed to a negative and printed, as is customary with 35-mm. film, but is developed by what is known as the *reversal* process. In this reversal process, the exposed image is first developed, and then the developed silver is dissolved in a *bleaching* bath, as it is called, which oxidizes the silver. This leaves behind the undeveloped silver bromide which was not affected by the developer, because it was not exposed to light. After a fresh exposure to light, this remaining silver bromide is developed in its turn and gives a positive. This is illustrated in Fig. 1, which is a drawing made from pictures taken through the microscope. In Section 1, we see the grains of the light-sensitive silver bromide in the emulsion, and in Section 2

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[1] Communication No. 378 from the Kodak Research Laboratories.

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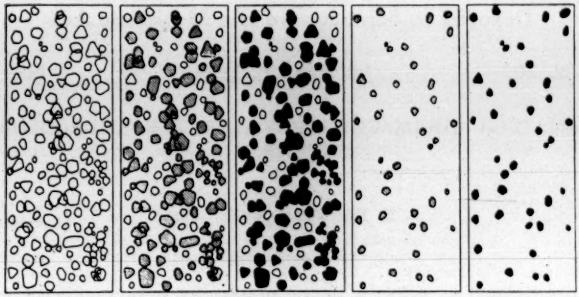
we have marked, by crosshatching them, those which have been affected by light during exposure in the camera. They would not show any change to the eye, of course, because the change by light is not visible. The grains form what is called the *latent image*. Now, after development, these exposed

FIG. 1.



Section 1 Section 2 Section 3 Section 4 Section 5

grains turn into black metallic silver, and this is shown in Section 3 of the diagram. Then the bleaching bath removes all the silver, and it leaves behind the silver bromide grains which were not exposed, as shown in Section 2. Now, these are re-exposed and developed and make the final positive, as we see in Section 5.

In this reversal process, control of the results is obtained by regulating the second exposure in proportion to the amount of exposure given in the camera. Thus, if in the camera a heavy exposure is given, much of the silver will be developed at the beginning, and when this is removed, there will be only a small amount of silver bromide left to be used for the production of the positive. It is necessary, therefore, to give a very heavy exposure in order to make all this developable and so get enough density in the image. On the other hand, if the camera exposure is light, there will be a great deal of undeveloped silver bromide available to form the final image, and it is necessary to give only a short exposure, as otherwise too dense an image will be produced. This reversal process

enables pictures of excellent quality, and with a very small amount of grain, to be obtained at low cost upon the narrow film so that the tiny pictures will stand the high magnification necessary for projection upon the screen.

Since 1923 this process has developed until it is now a considerable industry. The early apparatus described in my previous paper has been supplemented by other forms of cameras and projectors, so that at the present time a wide range of extremely convenient apparatus made by several different manufacturers is available for the public. As a general rule, at the present time, the cameras are driven by spring motors which are wound up between exposures, and the trend of design is in the direction of cameras as small and compact as possible, so that an amateur motion picture camera is little more bulky than the very small and compact cameras used for still photography.

The processing is now done on automatic machines consisting of a number of tanks which carry racks with rollers, by means of which the film is fed continuously through the solutions. The machines are entirely automatic, the films being fed in as they come from the customer and taken out of the drying cupboard as positives ready for projection. As the film travels down the machine, it is first developed to a negative, the developed silver is then removed in a bleaching bath, the film is cleared of the bleach and resensitized, and it is then exposed to an extent dependent upon the original exposure and controlled by the optical density of the film itself. The control is effected by the passage of the film between a source of red light and a thermopile. The current from the thermopile, produced by the heating effect of the red light passing through the film, controls a galvanometer vane, which is interposed in an optical system by means of which a beam of white light is projected upon the film. Thus, the second exposure is dependent upon the current in the thermopile and, therefore, upon the transmission of the red light by the film.

After this second exposure, the new image is developed as a positive, the film is fixed, washed, and dried; all these operations going on as the film travels forward through the machine. In a little more than an hour from the time the

film enters the machine, it is ready for projection. Each machine takes a new film every five minutes.

The processing stations are available all over the world, these containing more than a hundred of the automatic machines that are described above. At the present time, the traveler can get his film processed in Rio de Janeiro or in Japan or in China as easily as he can in Florida. In a few days, the pictures ready for projection are returned to him from the station.

From these original films, duplicates are now made by the same reversal process. A print is made in a small contact printer upon a fine-grained positive film prepared for the purpose, and this is then developed by the reversal process in an automatic machine, so that, from the original positive, any number of prints can be made in the same way and with the same ease as prints are made from a negative.

In photography it is, of course, more rational to use a positive than a negative as an original since the negative is only a means to an end and is of no value in itself. The earliest photographic processes gave positives, and when a process was first introduced in which a negative was obtained which had to be printed in order to get a final result, it was regarded as a defect in the process that it was unable to give a direct photograph; a defect, of course, which was more than compensated by the ease with which copies could be multiplied. Provided, as is the case by the use of the reversal process, that prints can be obtained from positives as easily as they can from negatives, the use of positives has the additional advantage that the original is of value as well as the prints made from it.

There have been many other developments in the field of amateur cinematography. Thus, at first, titles were made by setting up the original material with a printing press and photographing it in a camera of the ordinary type. By the use of typewritten titles and special cameras enabling the pictures to be made at high speed, the cost of titles has been very greatly reduced and their use enormously increased, so that at the present time it is very easy for an amateur to insert appropriate titles in his pictures and thus give them a more permanent value for projection in his absence. The

greatest development in the whole field of amateur cinematography lies in the increasing use of the 16-mm. for the projection of pictures originally made in the standard size. Very soon after the small size was introduced on the market, a library was formed in which reduction prints from standard professional negatives were available for renting and thus could be shown in the home. This was later supplemented by the supply from standard short-length negatives of prints which could be bought by the owners of projectors, and such prints, made by many different producers, are now available on the market.

The availability of the small film has made it possible to plan the use of films for teaching purposes on a much larger scale than could be considered if the expensive standard width film were the only one available, and complete programs of class room films for the instruction of children are now in course of preparation. These films have been designed by teachers collaborating with those still engaged in scholastic work throughout the country and are made to fit into the curriculum of the elementary schools and high schools. An experiment in this field has been conducted by the Eastman Kodak Company. This consisted in the production of a number of special films adapted to class room instruction. These films were then shown to groups of children in a number of schools in twelve cities in different parts of the country, groups in other schools in the same city being taught by equally competent teachers without the films.

The results obtained in this experiment show that the children who received instruction with the use of film made much greater progress in a series of objective tests than the children covering the same area of instruction but without the use of the film. Moreover, these results show that those pupils who had instruction with the film manifested greater interest in the work of the class room; that this interest was a sustained one extending through the school term; that these children did more reading of a finer quality; that their self-activities were stimulated; that they discussed and wrote more freely on the topics studied; that they correlated features of the film with their own personal experiences and with community life. The results achieved, therefore, through the

use of films, were the outstanding objectives of class room work.

As an instance of one of the many ramifications which the development of the amateur standard film has made possible, there may be mentioned an interesting application invented by Mr. George L. McCarthy and placed on the market, by the Eastman Kodak Company, under the name of the *Recordak*. Mr. McCarthy conceived the idea that instead of listing the details of the checks passing through a bank every day, it would be feasible to photograph them, the checks

FIG. 2.

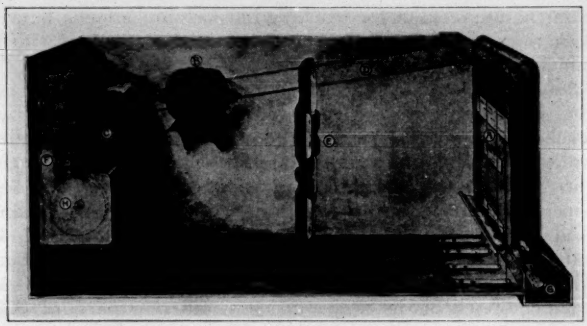


Diagram of Recordak.

being carried on an automatic belt in front of a camera which would record two checks on every frame of the amateur standard film. A hundred feet of film have four thousand frames and can thus record eight thousand checks. Another advantage of doing this is to provide a bank with a definite proof of the existence of each check drawn by a depositor before it is returned to the depositor, so that, should it be claimed later that the bank had made an error and had entered against a customer's account a check which did not exist, the photograph of the check could be produced as proof of the accuracy of the record. The Recordak instrument designed for this purpose is a very ingenious machine (Fig. 2). The checks are simply fed into a hopper, and as the amount

of one is tabulated on an adding machine connected to the Recordak, its predecessor passes into the machine and is photographed, being held on a vertical belt by wires passing over the face. The camera is double and will make either one or two records of the check, as may be desired, since in most cases it is necessary for the bank to send one set of records to another bank and retain one set for itself. The great economy of time and labor which is obtained by this new introduction is causing Recordaks to be installed in the banks as rapidly as they can be made available, and it is probable that the Recordak will play as important a part in the machinery of the banking business as the adding machine, thus introducing what may truly be called *accounting by photography.*

From the very beginning of photography, it has been the dream of every worker to produce pictures in natural colors instead of in monochrome, and in this field the Kodak Company has been experimenting for more than twenty-five years. Every process that appeared to be promising has been studied very carefully, but no process was considered to fulfill the requirements which were held by the officials of the company to be necessary for a process of amateur color photography. These requirements arose from the fundamental idea on which the Kodak Company is based. When Mr. Eastman started his work on photography, he was impressed by the difficulty that photography presented to the ordinary unskilled person; and the idea that photography should be accessible to anybody who was interested in obtaining photographs, whether he wished to acquire expert knowledge or not, led to the design of portable, flexible film and and the film cameras which have become so widely known as *Kodaks.* The same ideas to which the origin of the company itself was due still held in 1923, when it was made possible for anybody without professional skill to take motion pictures as easily as he had previously been able to take still pictures. In the search for a process of color photography, the same conception prevailed, and in the summer of 1928 it was felt that a process had been worked out which would meet those requirements.

The processes of color photography depend upon the fact first demonstrated in 1861 by Clerk Maxwell in a lecture at

the Royal Institution that colors can be duplicated in photography by taking advantage of the fact that any color can be matched by a mixture of three primary colors—red, green, and blue-violet. Maxwell took three photographs of a colored ribbon—one through a red solution, another through a green solution, and a third through a blue—and from the three negatives he made three lantern slides and projected them on the same screen in register from three lanterns, each one projecting its slide through the solution which had originally been used for taking the negative. From these three primary colors—red, green, and blue-violet—in the correct proportions, any color can be obtained. Thus, if red light is thrown on the screen and green light added to it, we shall get first orange-red and then orange, which, as the intensity of the green grows in proportion to the red, finally becomes yellow. Similarly, the addition of green to blue-violet produces shades of blue-green and finally of pure bluish green exactly opposite to the red sensation in color, while mixtures of red and blue produce all possible shades of violet, ranging through magenta into purple. Since the three colors together generate white when they are mixed in equal proportion, we can dilute any of the strong colors—red, yellow, orange, green, blue-green, or blue-violet—by the addition of one or two other colors.

In 1869, a little book in French by Louis Ducos du Hauron was published under the title *Les Couleurs en Photographie.* In this, du Hauron explained the analysis of light into three primary colors and proposed the obtaining of three negatives through three color filters, but, instead of proposing that the pictures should be projected on top of one another, he suggested two other methods of combining the colors to obtain a finished picture. One of these processes has since become of very considerable practical importance in color photography and is now known as the *screen plate* process. Du Hauron suggested that the surface of a glass plate or film might be covered with tiny unit filters and a sensitive emulsion coated on top of this. Exposures are made through the color filters, so that the image is cut up into a lot of little sections—so small that they are invisible to the eye, each section being taken through one of the filters. If the emulsion is so treated

in development that a positive is obtained, this will give a direct photograph in colors. This process was realized on a commercial scale in the Lumière Autochrome plate, which was first put on the market in 1907.

Another process which was discussed at greater length by du Hauron is that which is now known as the *subtractive* process. When a color picture is projected on the screen by means of a three-color projection lantern, the pictures made in this way, by adding three different lights, are said to be made by the *additive* process. Suppose that over a sheet of yellow gelatin, which will absorb blue light, is put a sheet of magenta gelatin, which will absorb green light; then, since the yellow absorbs the blue light, and the magenta, the green light, only red light is transmitted. A red image can thus be obtained either by projecting it through a red filter or by putting a magenta image on the top of a yellow one. In the same way, a green image can be obtained by putting a blue-green one on top of a yellow one, when the yellow will cut the blue out of the blue-green and leave only green.

In working the subtractive process, the three negatives are taken just as for the additive process, and then positives are printed in some way which enables them to be made of a colored material, the commonest being to make them by printing in bichromated gelatin. By this process, the three negatives can be printed in colored dye, the picture taken through the red filter being printed on gelatin dyed blue-green; the one taken through the green filter, on gelatin dyed magenta; and the one taken through the blue filter, on gelatin dyed yellow; and, now, if the three are cemented together in register, the resulting transparent color picture will reproduce the colors of the original subject.

We thus see that there are three distinct processes of color photography which must be borne in mind in any consideration of the subject:

(1) The additive processes, in which the three positives are projected through their corresponding filters on to a screen.

(2) The screen plate processes, in which the unit filters are so small that they are invisible but in which the process is still essentially an additive one.

(3) The subtractive processes, in which the filter negatives are printed in their complementary colors, and these prints are then superimposed.

For motion picture work, the additive processes would appear to present great advantages, but they have one serious disadvantage: they require a modification of the projection apparatus, and this has hindered their introduction into the motion picture theater field. The very beautiful Gaumont process of color motion picture photography, for instance, involved the taking of three pictures for every frame. These were taken one above the other through three lenses fitted with three filters and were then projected through a triple lens also fitted with filters, the three pictures being superposed on the screen to give a picture in color.

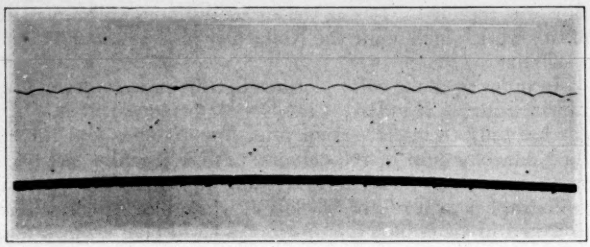
The processes which are in use at the present time for the production of colored motion pictures for the theaters are subtractive processes, and they usually take advantage of the fact that a film has two sides, so that one picture can be put on one side, and another, in register, on the other side of the film. We ourselves experimented with processes of this kind and have obtained some excellent results from a process which we term Kodachrome. While these two-color processes represent a practical compromise which may be useful in connection with the motion picture theater, the quality of the results is not satisfactory for outdoor scenes, and both the taking and the printing is too complicated for use by amateurs.

A most satisfactory process for amateur cinematography in colors would be the use of a screen film in which the film would carry filter units preferably dispersed regularly in dots or lines. There are, however, great practical difficulties in the preparation of such a screen and in securing the adherence of the emulsion to it, which have hitherto prevented such a process from attaining practical realization.

In 1908, R. Berthon patented a process which realized most of the advantages of a screen film process without involving too great difficulty in making the film. In this process, the color filters are placed in the lens, while the film is embossed on the support side with a number of small lenses. The film

therefore looks as is shown in Fig. 3. In this figure, the corrugated edge represents the surface of the film base, in which the small lenses shown have been embossed. We see below this the thickness of the base itself, and then a thick black line which represents the thickness of the emulsion. When this film is put into the camera, the embossed lenses will form miniature images of the filters on the film emulsion. Suppose, for instance, that the lens contains a stop with three holes in it—one covered by a red filter; a second, by a green; and a third, by a blue filter. Then, behind each lens there would be formed on the emulsion a group of three dots—one

FIG. 3.



Section of Kodacolor film.

dot corresponding to the red filter; one, to the green; and the third, to the blue filter. In this way, the effect produced by the embossed film and the filters in the lens is exactly similar to that which would be obtained by the use of a screen film in which the filter units were in contact with the emulsion, but the process has the advantage that the film is much easier to make and to use. The lenses can be either spherical or cylindrical. If they are cylindrical, the filters used in the lens can be strips parallel to the embossing on the film, and we shall then get line images of them on the film.

Some years after this invention, Berthon became associated with Anthon Keller-Dorian, the owner of an engraving plant in Alsace, and the Société Keller-Dorian-Berthon was formed to develop the process for commercial purposes. In 1925 this process was offered to the Eastman Kodak Company

and was thought to offer possibilities of success, especially as it was closely akin in its requirements to the method of amateur cinematography which the company had already developed. Rights in the process were therefore purchased, and development was started with a view to perfecting the process for use with the amateur 16-mm. film.

A great deal of study was involved. It was necessary to standardize the methods of making the lenses on the film; to design and make a suitable emulsion strongly sensitive to green and red light and yet with sufficiently fine grain for the minute structure of the separate color elements to be resolved; and, especially, to work out suitable methods by which the film could be developed and reversed while the rendering of color was retained. It was the successful accomplishment of all this work which made the Kodacolor process a commercial possibility.

In order to make color pictures by this process, a photographer using a standard Ciné-Kodak fitted with an f/1.9 lens has only to insert a color filter into the lens and thread the Kodacolor film in the camera. After the film has been exposed, it is sent away for processing and comes back to the photographer as a roll of black and white film which can be projected in an ordinary projector and will give a black and white picture on the screen. But if the projector is fitted with a special color filter like that which is used in the camera, then a colored picture will be obtained on the screen.

The color filter is composed of three separate areas—red, green, and blue—and it slips into the lens in the place of the ordinary hood, which is removable.

But the secret of the Kodacolor process is in the film. The film surface is embossed by running it through steel rollers with tiny cylindrical lenses composed of the film base material and extending lengthwise of the film. The lenses on the film are about four times narrower than the dots making up the illustrations in this journal and they are therefore invisible except under a microscope. They cover completely the surface of the side of the film opposite from the sensitive emulsion. That surface faces the camera lens, and the emulsion is away from the lens (see Fig. 3).

When the trigger of the camera is pressed, light reflected

from the subject passes selectively through the three-color filter, on through the camera lens, and thence through the tiny embossed lenses on the film to the sensitive emulsion coating on the opposite side, where it is recorded (Fig. 4).

FIG. 4.

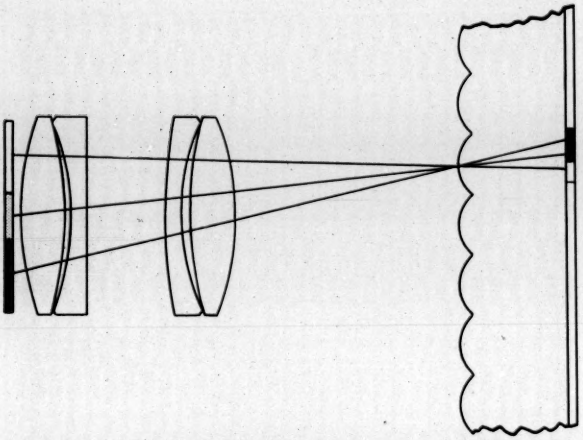
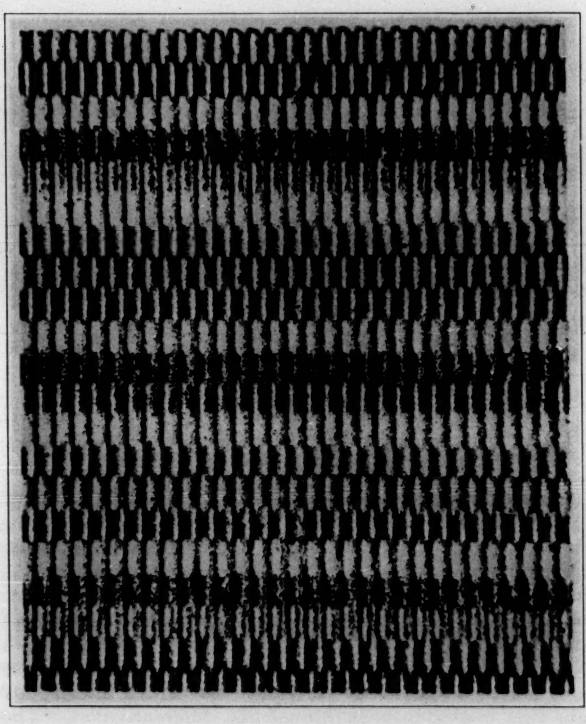


Diagram of Kodacolor process.

The function of the lenses embossed on the film is to guide the rays of light falling upon each tiny area and lay them on the sensitive emulsion as three distinct impressions corresponding to the three filter areas, so that the three colors covering the lenses are imaged behind each tiny cylindrical lens as three parallel vertical strips, because the tiny cylindrical lenses are parallel to the strips of color on the filter. Thus, the width of each of the minute areas of emulsion is subdivided into three parts related to the three filter areas and affected by light that is able to pass through the different colors. The sum of these invisibly small affected areas of film constitutes the whole photographic image. In Fig. 5 is shown a photomicrograph of a picture showing a number of colored strips running horizontally across the image. The vertical strips are produced by the embossed lenses.

A red ray from an object in front of the camera, for instance, reaches the sensitive material of the film at a spot related to the red area of the filter. The *reversal process*

FIG. 5.



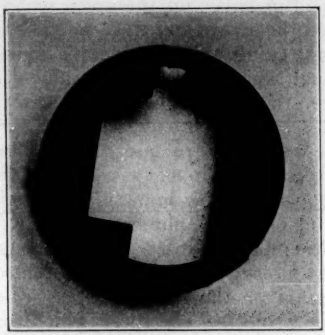
Photomicrograph of Kodacolor picture of narrow colored strips. The strips are horizontal, the embossed lenses running vertically.

turns this affected spot into a transparent area, leaving opaque the adjoining, unaffected areas related to the green and blue segments of the filter.

So, also, with the green and blue and with combinations of colors. The sum of the points on the scene containing red makes a photograph from red light on the emulsion areas related to the red filter area; the sum of the blue also makes a separate photograph; and, similarly, the green.

It is necessary to have some method of controlling the exposure for the separate colors. The film may vary somewhat in its sensitiveness. Even if we make one batch of film

FIG. 6.



Ratio diaphragm cap.

which has equal sensitiveness to the three different filters, it does not follow that this can be duplicated exactly, and on the exactness of adjustment depends the color rendering. This adjustment for the different sensitiveness of the emulsions is accomplished by cutting down the length of the color filters used in the camera. This is done by what is termed a *ratio diaphragm cap* (Fig. 6). These caps are chosen, after the emulsion has been tested, to correspond to the color sensitiveness of that particular batch of emulsion, and the cap is then placed on the end of the film in such a position that it cannot be missed by the user. All that is necessary is for him to take out his color filter from the camera when placing a new film in the camera, throw away the old cap, and replace it by a new one from the end of the fresh film.

In this way, he can always be sure of getting the correct ratio of exposures.

Now, in order to project the pictures, the developed film is put in the projector, which contains exactly the same optical system reversed. Behind the film are the condenser and the source of light. The color filter consists of the same three primary colors—red, green, and blue. The reverse of the fact that white light divides up into the colors of the spectrum is that light coming evenly out from the three colors of the filter on a projector and superimposed on a screen appears white. But cover up the green and blue segments of the filter, and the screen will turn red. Cover up the red and blue, and the result will be green. Varying the areas of each color through which the light may shine will give infinite shadings between these colors. Black is the total elimination of light.

Now, when we have a picture on the film, the opaque areas of the film cover up, in effect, certain of the filter areas; they prevent the light from going through where it is not needed by cutting off, at the film, rays which would otherwise pass through the embossed lenses, through the projecting lens, and through the filter area in question to the screen.

From any point on the scene, the only colors which are permitted to be projected are those which, on the screen, blend into the corresponding original colors of the scene photographed. The pattern of these rays, from all the cylindrical lenses on each frame, projects a picture on the screen, with each ray contributing its speck of light to the color or blend of colors at one point.

Owing to the absorption of the light by the color filter used in projection, it is necessary to be content with a small picture on the screen, and for this reason small screens are used, so that the illumination can be sufficient.

As will be seen, the projector system is optically an exact inversion of the camera system, but there is in general one difference between the camera and the projector: the focal length of the projection lens is usually considerably greater than that of the camera lens. The relative apertures must be the same; that is, the cone of light coming through the projector lens must have exactly the same angle as the cone

of light coming through the camera lens, since otherwise the optical conditions will not be reproduced correctly and the colors will not be correct. A difference in focal length can, however, be compensated by the use of a supplementary lens, which places images of the color filters at the right point in the optical system; and this lens, which is termed a *compensator*, is fitted on to the lens of the projector. The *compensator* makes no difference when black and white pictures are being projected except to change slightly the focal length of the projection lens, but is required to obtain an even screen when Kodacolor pictures are being projected.

Although the Kodacolor process has been on the market for only a short time, the results which are being obtained by the users show quite definitely that it is successful. The results are, on the whole, excellent, and provided that the photographer exercises reasonable care in the choice of his subjects and in the operation of the camera, there is no more difficulty at the present time in obtaining motion pictures in color than in the making of still photographs.

ROCHESTER, NEW YORK,
October 19, 1928.

Botanical Orders and Chemical Composition. THOMAS PERCY HILDITCH of the University of Liverpool (*Proc. Royal Soc.*, B, 1928, *103*, 111–117) notes a marked tendency toward the production of quite specific fatty acids in the seed fats of certain orders of plants. Thus the *Palmae* or palms produce lauric acid; the *Myristiceae* or nutmeg family produce myristic acid; the *Cruciferae* or cress or mustard family produce erucic acid; the *Umbelliferae* or parsley family and the closely related *Ariliaceae* or English ivy family produce petroselinic acid, which is a metamer of oleic acid and has its double bond between the sixth and the seventh carbon atoms from the carboxyl group. Oleic, linoleic, palmitic, arachidic, lignoceric, and other acids may also be present as minor constituents of the fats.

J. S. H.

Preservation of Peaches. CHARLES W. CULPEPPER, JOSEPH S. CALDWELL, and R. C. WRIGHT of the Bureau of Plant Industry (*U. S. Dep. Agric. Technical Bull.* No. 84, 1–13, 1928) have published the results of a research of three years' duration on the preservation of peaches with the retention of their flavor so that the product could be used in the manufacture of ice cream.

Peaches of suitable varieties and of the proper degree of ripeness were either crushed or pulped. The resulting material was divided into three lots, to two of which were added 10 per cent. and 40 per cent., respectively, by weight of sugar, while the third lot contained no added sugar. Each lot was divided into two portions, one of which was canned, the other frozen. The process of canning was conducted in the usual manner, but enamel cans were found most suitable. For preservation by cold storage, the uncooked material was placed in hermetically sealed cans which were then kept at a temperature of 15° F. until their contents had frozen; they were then kept at a temperature of 15° to 25° F. until used. Cold storage at a temperature of 32° F. was not satisfactory. The chief advantage in the addition of sugar was the prevention of fermentation of the raw fruit prior to complete freezing of the contents of the cans; addition of 40 per cent. by weight of sugar was found preferable. Peaches preserved by each of these processes kept in good condition for periods as long as 21 months. They retained their value as a flavor for ice cream. Freezing or cold storage gave a slightly better preservation of the fresh fruit flavor, while canning had the advantage of a lower cost of production and distribution.

J. S. H.

THE SCATTERING OF A STANDING ELECTRO-MAGNETIC WAVE.

BY

MILDRED ALLEN, Ph.D.

Bartol Research Fellow.

BARTOL RESEARCH FOUNDATION

Communication No. 31.

Introduction. During consideration of the pure Compton effect, the question arose as to what would happen if instead of the running electromagnetic wave the electron were to encounter a standing electromagnetic wave. Would the scattered radiation be the sum of that expected from the two component waves or would the effect be more complicated? An answer to this question has been attempted on the basis of the Schroedinger wave mechanics; it is dealt with essentially from the point of view put forth by Gordon,[1] but in addition the simple perturbation theory given by Klein [2] is shown to yield the same solution, if carried out to its ultimate conclusion. The Schroedinger theory has not yet been developed to the point where it is utterly consistent in telling the whole story of the Compton effect, but in carrying out the computations in connection with the standing electromagnetic wave we shall try to make the procedure as consistent as possible and still retain the theory in its present state.

The procedure for the simpler case of a running wave may be sketched as follows. (1) The Schroedinger wave equation for an electron in an electromagnetic field is derived by inserting the appropriate Hamiltonian operator in the diffusion equation. (2) A solution ψ_s is found for this differential equation for the case in which the vector and scalar potentials involved are those of a plane X-ray wave and where the amplitudes of these potentials are assumed so small that terms involving their squares are to be neglected. The constants appearing in this solution are correlated with

[1] Gordon, W.: *ZS. f. Physik*, 40, 117, 1926.

[2] Klein, O.: *ZS. f. Physik*, 41, 407, 1927.

the energy and momentum of the electron. For each value of the momentum $\mathbf{p}$ of the electron the simple solution is to be multiplied by a normalizing factor $C(p)$. We must now find in terms of these individual solutions one solution ψ which at $t = 0$ satisfies given initial conditions. Since the momentum of the electron may vary continuously from minus infinity to plus infinity, such a ψ will take the form of the integral

$$\psi = \int_{-\infty}^{\infty} \psi_s C(p) z(p) dp, \tag{1}$$

where $dp = dp_x dp_y dp_z$ and where ψ_s corresponds to the solution for constant p_x, p_y, p_z. This integral thus represents an infinite sum in these individual solutions with coefficients $z(p)$ which are to be determined to satisfy these initial conditions. (3) We then build up from this value of ψ and from its conjugate $\bar{\psi}$ the expression for the effective current density which in terms of ψ and $\bar{\psi}$ has been given by Gordon and Klein as

$$\rho \mathbf{u} = \frac{e}{2m_0} \frac{h}{2\pi i} \left(\bar{\psi} \operatorname{grad} \psi - \psi \operatorname{grad} \bar{\psi} - \frac{4\pi i}{h} \frac{e}{c} \mathbf{U} \psi \bar{\psi} \right). \tag{2}$$

(4) From this value of the current density we construct the vector potential $\mathbf{U}'$ which appears as an integral with respect to p_x, p_y, p_z, each element of the integral being characterized by the particular values of p_x, p_y, p_z, by $h\nu$, by the unit vector $\mathbf{n}$ in the direction of the incident radiation, and by the unit vector $\boldsymbol{\xi}$ in the direction of observation. The coefficient of the time in the exponent is of the form $h\nu_{1P}$ where

$$h\nu_{1P} = E - E_{1P} + h\nu, \tag{3}$$

and E_{1P} is a certain definite function of the p's, $\mathbf{n}$ and $\boldsymbol{\xi}$. If now we consider a problem in which an electron is permitted to have only two stationary states characterized by p_x, p_y, p_z, E and by p_{x1}, p_{y1}, p_{z1}, E_1, the Schroedinger theory for the problem gives for the vector potential of the emitted radiation an expression in which the coefficient of t is equal to $h\nu_1$ where

$$h\nu_1 = E - E_1 + h\nu. \tag{4}$$

It gives only one direction in which there is any intensity for

this radiation, the direction being characterized by the unit vector ξ_1 given by the vector equation

$$\mathbf{p}_1 - \mathbf{p} = \frac{h\nu}{c}\mathbf{n} - \frac{h\nu_1}{c}\xi_1. \tag{5}$$

Now in this problem the quantities $\mathbf{p}_1$, E_1, ξ_1 occur in the expression for $\mathbf{U}'$ in identically the same positions as do certain quantities E_{1P}, $\mathbf{p}_{1P}$ and ξ_1 (which are related by an equation of the type (5) and by the relation $|\mathbf{p}_{1P}| = \sqrt{\frac{E_{1P}^2}{c^2} - m_0^2c^2}$) in the expression for $\mathbf{U}'$ for the general problem. Our hypothesis of correlation is therefore to say that in that problem a quantum of energy of frequency ν_{1P} given by

$$h\nu_{1P} = E - E_{1P} + h\nu \tag{6}$$

may be radiated in a direction given by ξ_1 satisfying the vector equation

$$\mathbf{p}_{1P} - \mathbf{p} = \frac{h\nu}{c}\mathbf{n} - \frac{h\nu_{1P}}{C}\xi_1, \tag{7}$$

that the probability of such an event taking place is dependent upon the coefficient of $e^{2\pi i\nu_{1P}(t-R/c)}$ in the expression for $\mathbf{U}'$, and that whenever the quantum is radiated it is accompanied by the emission of an electron of energy equal to the quantity E_{1P} and of momentum equal to the quantity $\mathbf{p}_{1P}$. The correlation procures conservation of energy as between the initial and final states of the electron and radiation, and it procures conservation of momentum provided that the momentum of the quantum is regarded as $\frac{h\nu}{c}$.

In applying this procedure to the case of a standing electromagnetic wave, the vector and scalar potentials for the sum of two equal and opposite plane X-ray waves must be used in the original differential equation for ψ. That part of the emitted vector potential which is dependent on the time then turns out to consist to a first approximation of two additive parts, one resulting from the component

X-ray wave travelling in one direction and the second from the other component wave travelling in the opposite direction. The function multiplying the frequency factor in each of these terms governs the probability that the X-ray concerned will be scattered by the given electron. The radiation observed in any direction then consists of two parts, differing in frequency and depending in intensity on the probability that the component wave will be scattered in that direction. The correlation with the motion of the recoil electron for each of these terms is carried out separately as indicated above in the more simple case.

1. *Derivation of the Schroedinger equation.* The fundamental equation of the Schroedinger wave mechanics, as it is accepted to-day, is the diffusion equation [3]

$$-\frac{h}{2\pi i}\frac{\partial \psi}{\partial t} = H\left(\frac{h}{2\pi i}\frac{\partial}{\partial q_r}, q_r\right)\psi. \tag{8}$$

H is here the Hamiltonian operator, which for the purposes in hand may be regarded as being derived from the form

$$H = \sum_r \dot{q}_r p_r - L, \tag{9}$$

where L is the quantity $L(\dot{q}_1, \dot{q}_2, \cdots q_1, q_2, \cdots t)$ which makes $\delta \int L dt = 0$ give the classical equations of motion. For an electron in an electromagnetic field having vector potential $\mathbf{U}$ and scalar potential φ, L is known to be given by the following equation:

$$L = -m_0 c^2 \sqrt{1 - \dot{q}^2/c^2} - e\varphi + \frac{e}{c}(\mathbf{U}\cdot\dot{\mathbf{q}}), \tag{10}$$

where the dot represents the scalar product of the two adjoining vectors. Since p_r may be defined as $\partial L/\partial \dot{q}_r$, we have for $\mathbf{p}$ the vector equation

$$\mathbf{p} = \frac{m_0 \dot{\mathbf{q}}}{\sqrt{1 - \dot{q}^2/c^2}} + \frac{e}{c}\mathbf{U} \tag{11}$$

and the important Hamiltonian function (9) may be expressed as

[3] Darwin, C. G.: *Roy. Soc. Proc.*, 117, 258, 1927.

$$H = \mathbf{p}\cdot\dot{\mathbf{q}} + m_0c^2\sqrt{1 - \dot{q}^2/c^2} + e\varphi - \frac{e}{c}(\mathbf{U}\cdot\dot{\mathbf{q}}) \qquad (12)$$

$$= \frac{m_0c^2}{\sqrt{1 - \dot{q}^2/c^2}} + e\varphi,$$

which shows H to be the sum of the kinetic and potential energies which it is so often assumed to be. H must, however, for use in (8) be expressed in terms of the momenta p_r. Equation (11) squared gives

$$\left(p_x - \frac{e}{c}U_x\right)^2 + \left(p_y - \frac{e}{c}U_y\right)^2 + \left(p_z - \frac{e}{c}U_z\right)^2$$

$$= \frac{m_0^2\dot{q}^2}{1 - \dot{q}^2/c^2} = m_0^2c^2\left(-1 + \frac{1}{1 - \dot{q}^2/c^2}\right), \qquad (13)$$

which solving for $1/\sqrt{1 - \dot{q}^2/c^2}$ gives the relation

$$\frac{m_0c^2}{\sqrt{1 - \dot{q}^2/c^2}} = c\left[\left(\mathbf{p} - \frac{e}{c}\mathbf{U}\right)^2 + m_0^2c^2\right]^{\frac{1}{2}}. \qquad (14)$$

When (14) is inserted in (12), H is expressed as a function of the p's and q's as required. For the H operator appearing in the diffusion equation (8) p_r is to be replaced by $(h/2\pi i)(\partial/\partial q_r)$ so that that equation becomes

$$-\frac{h}{2\pi i}\frac{\partial\psi}{\partial t} = c\left[\left(\frac{h}{2\pi i}\,\mathrm{grad} - \frac{e}{c}\mathbf{U}\right)^2 + m_0^2c^2\right]^{\frac{1}{2}}\psi + e\varphi\psi, \qquad (15)$$

and this is the fundamental equation for the ψ associated with an electron in an electromagnetic field. Expanding, (15) becomes

$$-\frac{h}{2\pi i}\frac{1}{c}\frac{\partial\psi}{\partial t} = \left[-\frac{h^2}{4\pi^2}\nabla\cdot\nabla - \frac{h}{2\pi i}\frac{e}{c}(\mathbf{U}\cdot\mathrm{grad} + \mathrm{div}\ \mathbf{U})\right.$$

$$\left. + \frac{e^2}{c^2}\mathbf{U}^2 + m_0^2c^2\right]\psi + \frac{e}{c}\varphi\psi. \qquad (16)$$

The operator acting on ψ may be applied twice, giving

$$\left(-\frac{h}{2\pi i}\frac{1}{c}\frac{\partial}{\partial t} - \frac{e}{c}\varphi\right)^2\psi = \left[-\frac{h^2}{4\pi^2}\nabla\cdot\nabla - \frac{h}{2\pi i}\frac{e}{c}(\mathbf{U}\cdot\mathrm{grad}\right.$$

$$\left. + \mathrm{div}\ \mathbf{U}) + \frac{e^2}{c^2}\mathbf{U}^2 + m_0^2c^2\right]\psi \qquad (17)$$

In view of the fact that

$$\text{div } \mathbf{U}\psi = \mathbf{U}\cdot\text{grad } \psi + \psi \text{ div } \mathbf{U} \tag{18}$$

and of the equation of continuity

$$\text{div } \mathbf{U} + \frac{1}{c}\frac{\partial \varphi}{\partial t} = 0 \tag{19}$$

equation (17) reduces to

$$\left[-\frac{h^2}{4\pi^2}\left(\nabla\cdot\nabla - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) - 2\frac{h}{2\pi i}\frac{e}{c}\left(\mathbf{U}\cdot\text{grad} + \frac{\varphi}{c}\frac{\partial}{\partial t}\right) + m_0^2c^2 + \frac{e^2}{c^2}(\mathbf{U}^2 - \varphi^2)\right]\psi = 0. \tag{20}$$

The diffusion equation now appears for the case of an electron in an electromagnetic field as the older and better known wave equation of Schroedinger. This may be seen to be identical with that found by Gordon (l.c. p. 119 equation 9).

2. *Solution of the Schroedinger equation.* The solution of this Schroedinger equation requires that we make assumptions as regards the form of the vector and scalar potentials. In the case of the ordinary Compton effect, we assume simple harmonic potentials with the frequency of the X-ray, i.e.

$$\begin{aligned} \mathbf{U} &= \mathbf{a}\cos\frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r} - ct), \\ \varphi &= a\cos\frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r} - ct), \end{aligned} \tag{21}$$

where the amplitudes are so small that their squares may be neglected. Then $\mathbf{U}^2$ and φ^2 are immediately to be neglected. For the case in which we are interested, that of a standing electromagnetic wave, we shall assume potentials of the form

$$\begin{aligned} \mathbf{U} = \mathbf{U}_1 + \mathbf{U}_2 &= \mathbf{a}\left[\cos\frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r} - ct) + \cos\left\{\frac{2\pi\nu}{c}(-\mathbf{n}\cdot\mathbf{r} - ct) + \delta\right\}\right], \\ \varphi = \varphi_1 + \varphi_2 &= a\left[\cos\frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r} - ct) + \cos\left\{\frac{2\pi\nu}{c}(-\mathbf{n}\cdot\mathbf{r} - ct) + \delta\right\}\right], \end{aligned} \tag{22}$$

where ν is the frequency of the X-ray, $\mathbf{n}$ a unit vector in the direction of this incident radiation, and δ an arbitrary constant phase angle. It is of interest to note that for the case of a plane electromagnetic wave, if $\mathbf{a}$ is taken in the direction of and proportional to the electric vector,[4] the scalar potential is necessarily zero, owing to the fact that the electric vector lies in the plane of the wave front and so is perpendicular to the direction of propagation $\mathbf{n}$. This is easily shown, for

$$\operatorname{div} \mathbf{U} = -\frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{a}) \sin\frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r} - ct)$$

which vanishes for $\mathbf{a}$ perpendicular to $\mathbf{n}$, in which case $(\partial\varphi/\partial t) = 0$. The electric vector is then $-(1/c)(\partial\mathbf{U}/\partial t)$ and has the direction of $\mathbf{U}$. Hence, since standing waves are necessarily made up of plane waves, φ in (22) may vanish for a proper choice of $\mathbf{a}$. But, however $\mathbf{a}$ may be chosen, terms involving squares of the amplitudes are small enough to be neglected.

To solve the differential equation (20) we may follow either Gordon or Klein. First, following Gordon, let us assume a solution of the form

$$\psi = e^{(2\pi i/h)(-Et+\mathbf{p}\cdot\mathbf{r})} e^{(2\pi i/h)\lambda} \tag{23}$$

where λ is a function of x, y, z and t, and p_x, p_y, p_z and E are constants satisfying the relation

$$p_x^2 + p_y^2 + p_z^2 - \frac{E^2}{c^2} + m_0^2c^2 = 0. \tag{24}$$

This gives us an equation in λ

$$\frac{h}{2\pi i}\left(\nabla\cdot\nabla\lambda - \frac{1}{c^2}\frac{\partial^2\lambda}{\partial t^2}\right) - 2\frac{e}{c}\left[(\mathbf{p} + \operatorname{grad}\lambda)\cdot\mathbf{U} + \left(-E + \frac{\partial\lambda}{\partial t}\right)\frac{\varphi}{c}\right] + 2\left[\mathbf{p}\cdot\operatorname{grad}\lambda + \frac{E}{c^2}\frac{\partial\lambda}{\partial t}\right] + \left[(\operatorname{grad}\lambda)^2 - \frac{1}{c^2}\left(\frac{\partial\lambda}{\partial t}\right)^2\right] = 0. \tag{25}$$

[4] Waller, I.: *Phil. Mag.*, 4, 1228, 1927.

As suggested by Gordon's results, let us assume λ to be of the form

$$\lambda = \lambda_1 + \lambda_2 = A_1 \sin\frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r} - ct) + A_2 \sin\left\{\frac{2\pi\nu}{c}(-\mathbf{n}\cdot\mathbf{r} - ct) + \delta\right\} \quad (26)$$

where A_1 and A_2 are coefficients the values of which are to be determined by the conditions laid upon λ by the differential equation.

A complete theory would assume such an expression for λ that $e^{(2\pi i/h)\lambda}$ would be equal to unity at $t = 0$, at which time the vector and scalar potentials would not yet be acting and so would be zero. Owing to the customary incompleteness of expression in problems of this kind λ is periodic and never disappears. Our best hope is to say that ψ is like (23), but with a slowly varying coefficient of λ which makes $\lambda = 0$ at $t = 0$ and then at $t = 0$

$$\psi = e^{(2\pi i/h)(-Et + \mathbf{p}\cdot\mathbf{r})}. \quad (27)$$

This is the value of ψ which the fundamental hypothesis of this theory associates with an electron with energy E and momentum $\mathbf{p}$, the association being rendered possible through the relation (24) which was required by the differential equation (20) and also demanded as the relation between the momentum and energy for the relativistic dynamics of the electron.

We must now show two things about our assumed solution (23): prove that it does satisfy the differential equation (20), and find the appropriate values of the constants A_1 and A_2. Because of the wave form assumed for λ, the d'Alembertian vanishes and our equation (20) becomes

$$2\frac{e}{c}\left[\mathbf{p}\cdot\mathbf{U}_1 + \mathbf{p}\cdot\mathbf{U}_2 + \text{grad}\,(\lambda_1 + \lambda_2)\cdot(\mathbf{U}_1 + \mathbf{U}_2) - \frac{E}{c}(\varphi_1 + \varphi_2) + \left(\frac{\partial\lambda_1}{\partial t} + \frac{\partial\lambda_2}{\partial t}\right)\frac{1}{c}(\varphi_1 + \varphi_2)\right]$$
$$- 2\left[\mathbf{p}\cdot\text{grad}\,\lambda_1 + \frac{E}{c^2}\frac{\partial\lambda_1}{\partial t}\right] - 2\left[\mathbf{p}\cdot\text{grad}\,\lambda_2 + \frac{E}{c^2}\frac{\partial\lambda_2}{\partial t}\right]$$

$$-\left[(\text{grad}\,\lambda_1)^2-\frac{1}{c^2}\left(\frac{\partial\lambda_1}{\partial t}\right)^2\right]-\left[(\text{grad}\,\lambda_2)^2-\frac{1}{c^2}\left(\frac{\partial\lambda_2}{\partial t}\right)^2\right]$$
$$-2\left[\text{grad}\,\lambda_1\cdot\text{grad}\,\lambda_2-\frac{1}{c^2}\frac{\partial\lambda_1}{\partial t}\frac{\partial\lambda_2}{\partial t}\right]=0. \quad (28)$$

In view of the fact that the sum of the squares of the direction cosines equals unity, the following expressions vanish:

$$(\text{grad}\,\lambda_1)^2-\frac{1}{c^2}\left(\frac{\partial\lambda_1}{\partial t}\right)^2=0,$$
$$(\text{grad}\,\lambda_2)^2-\frac{1}{c^2}\left(\frac{\partial\lambda_2}{\partial t}\right)^2=0. \quad (29)$$

The equation may now be expressed, inserting the values of $\mathbf{U}$ and φ from (22), as the sum of terms in $\cos(2\pi\nu/c)(\mathbf{n}\cdot\mathbf{r}-ct)$, of terms in $\cos\{(2\pi\nu/c)(\mathbf{n}\cdot\mathbf{r}-ct)+\delta\}$ and of other terms:

$$\cos\frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r}-ct)\left[\frac{e}{c}\left(\mathbf{p}\cdot\mathbf{a}-\frac{E}{c}a\right)-A_1\frac{2\pi\nu}{c}\left(\mathbf{p}\cdot\mathbf{n}-\frac{E}{c}\right)\right]$$
$$+\cos\left\{\frac{2\pi\nu}{c}(-\mathbf{n}\cdot\mathbf{r}-ct)+\delta\right\}\left[\frac{e}{c}\left(\mathbf{p}\cdot\mathbf{a}-\frac{E}{c}a\right)\right.$$
$$\left.+A_2\frac{2\pi\nu}{c}\left(\mathbf{p}\cdot\mathbf{n}+\frac{E}{c}\right)\right]+\frac{e}{c}\left[\text{grad}\,(\lambda_1+\lambda_2)\cdot(\mathbf{U}_1+\mathbf{U}_2)\right.$$
$$\left.+\frac{1}{c}\left(\frac{\partial\lambda_1}{\partial t}+\frac{\partial\lambda_2}{\partial t}\right)(\varphi_1+\varphi_2)\right]$$
$$-\left[\text{grad}\,\lambda_1\cdot\text{grad}\,\lambda_2-\frac{1}{c^2}\frac{\partial\lambda_1}{\partial t}\frac{\partial\lambda_2}{\partial t}\right]=0. \quad (30)$$

In order that the coefficient of $\cos(2\pi\nu/c)(\mathbf{n}\cdot\mathbf{r}-ct)$ should vanish, the coefficient A_1 must have the value

$$A_1=\frac{e\left(\mathbf{p}\cdot\mathbf{a}-\frac{E}{c}a\right)}{2\pi\nu\left(\mathbf{p}\cdot\mathbf{n}-\frac{E}{c}\right)}. \quad (31)$$

Similarly, so that the term in $\cos\{(2\pi\nu/c)(-\mathbf{n}\cdot\mathbf{r}-ct)+\delta\}$ may vanish, we have

$$A_2 = -\frac{e}{2\pi\nu}\frac{\left(\mathbf{p}\cdot\mathbf{a} - \frac{E}{c}a\right)}{\left(\mathbf{p}\cdot\mathbf{n} + \frac{E}{c}\right)}. \tag{32}$$

The remaining terms all contain as a factor the product of a λ derivative with a vector or scalar potential or with another λ derivative, and in view of the fact that A_1 and A_2 both consist of terms proportional to the amplitudes of the electromagnetic potentials, are all terms of the order already neglected in deriving (25). Consequently we have for the solution of the diffusion equation for an electron hit by a standing X-ray wave, the wave being defined by the vector and scalar potentials (22), the expression

$$\psi = e^{(2\pi i/h)(-Et + \mathbf{p}\cdot\mathbf{r})} e^{(2\pi i/h)\lambda} \tag{33}$$

subject to the condition

$$\mathbf{p}^2 - \frac{E^2}{c^2} + m_0^2 c^2 = 0, \tag{24}$$

where

$$\lambda = \frac{e}{2\pi\nu}\frac{\left(\mathbf{p}\cdot\mathbf{a} - \frac{E}{c}a\right)}{\left(\mathbf{p}\cdot\mathbf{n} - \frac{E}{c}\right)}\sin\frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r} - ct)$$

$$-\frac{e}{2\pi\nu}\frac{\left(\mathbf{p}\cdot\mathbf{a} - \frac{E}{c}a\right)}{\left(\mathbf{p}\cdot\mathbf{n} + \frac{E}{c}\right)}\sin\left\{\frac{2\pi\nu}{c}(-\mathbf{n}\cdot\mathbf{r} - ct + \delta\right\}. \tag{33a}$$

This may be written more simply as

$$\psi = e^{(2\pi i/h)(-Et + \mathbf{p}\cdot\mathbf{r})} e^{(2\pi i/h)(A_1 \sin\theta_1 + A_2 \sin\theta_2)}, \tag{34}$$

where

$$\theta_1 = \frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r} - ct) \quad \text{and} \quad \theta_2 = \frac{2\pi\nu}{c}(-\mathbf{n}\cdot\mathbf{r} - ct) + \delta.$$

Alternative solution. This simple solution of (20) may also be arrived at by a direct application of perturbation theory

as indicated by Klein. In that case to emphasize the smallness of the amplitudes and to have $\mathbf{U}$ and φ in the more convenient exponential form, we shall express (22) as

$$\begin{aligned} \mathbf{U} &= \sigma\mathbf{a}'(e^{i\theta_1} + e^{-i\theta_1} + e^{i\theta_2} + e^{-i\theta_2}), \\ \varphi &= \sigma a'(e^{i\theta_1} + e^{-i\theta_1} + e^{i\theta_2} + e^{-i\theta_2}), \end{aligned} \tag{22a}$$

where θ_1 and θ_2 have the same significance as before, and σ is a small parameter such that terms containing its square are to be neglected. a' is seen to be $a/2\sigma$ and a similar relation holds for $\mathbf{a}'$. In solving an equation from the point of view of perturbation theory, we seek a solution of the perturbed equation in terms of the unperturbed solutions, i.e. in this case in terms of solutions of the equation in which $\mathbf{U} = \varphi = 0$, which we have seen to be satisfied by

$$\psi = e^{(2\pi i/h)(-Et + \mathbf{p}\cdot\mathbf{r})}. \tag{27}$$

Our problem is then to get a solution of (20) which is not far different from (27). Let us assume a solution of the form $\psi + \sigma f$ where ψ is given by (27). Making this substitution and neglecting terms in σ^2 the equation (20) becomes

$$\sigma\left[-\frac{h^2}{4\pi^2}\left(\nabla\cdot\nabla f - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}\right) + m_0^2c^2 f\right] = 2\frac{h}{2\pi i}\frac{e}{c}\left(\mathbf{U}\cdot\operatorname{grad}\psi + \frac{\varphi}{c}\frac{\partial\psi}{\partial t}\right), \tag{35}$$

or inserting the values of the potentials

$$-\frac{h^2}{4\pi^2}\left(\nabla\cdot\nabla f - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}\right) + m_0^2c^2 f = 2\frac{e}{c}\left(\mathbf{a}'\cdot\mathbf{p} - \frac{E}{c}a'\right)e^{(2\pi i/h)(-Et + \mathbf{p}\cdot\mathbf{r})}[e^{i\theta_1} + e^{-i\theta_1} + e^{i\theta_2} + e^{-i\theta_2}].$$

As indicated by Schroedinger,[5] we shall assume f to be of the form

$$f = w_+(x, y, z)e^{-(2\pi i/h)(E + h\nu)t} + w_-(x, y, z)e^{-(2\pi i/h)(E - h\nu)t} \tag{36}$$

and equate the resulting coefficients of $e^{-(2\pi i/h)(E+h\nu)t}$ and $e^{-(2\pi i/h)(E-h\nu)t}$ on the two sides of the equation. This leads to two separate equations

[5] Schroedinger: *Ann. d. Physik*, 81, 115, 1926.

$$-\frac{h^2}{4\pi^2}\nabla\cdot\nabla w_+ - \frac{(E+h\nu)^2}{c^2}w_+ + m_0^2c^2w_+$$
$$= \frac{2e}{c}\left(\mathbf{a}'\cdot\mathbf{p} - \frac{E}{c}a'\right)e^{(2\pi i/h)\mathbf{n}\cdot\mathbf{r}}\left(e^{(2\pi i/h)(h\nu/c)\mathbf{n}\cdot\mathbf{r}} + e^{-(2\pi i/h)(h\nu/c)\mathbf{n}\cdot\mathbf{r}+i\delta}\right)$$
$$-\frac{h^2}{4\pi^2}\nabla\cdot\nabla w_- - \frac{(E-h\nu)^2}{c^2}w_- + m_0^2c^2w_-$$
$$= \frac{2e}{c}\left(\mathbf{a}'\cdot\mathbf{p} - \frac{E}{c}a'\right)e^{(2\pi i/h)\mathbf{p}\cdot\mathbf{r}}\left(e^{-(2\pi i/h)(h\nu/c)\mathbf{n}\cdot\mathbf{r}} + e^{(2\pi i/h)(h\nu/c)\mathbf{n}\cdot\mathbf{r}-i\delta}\right). \quad (37)$$

To eliminate the remaining exponentials, let

$$\begin{aligned} w_+ &= B_1e^{(2\pi i/h)[\mathbf{p}+(h\nu/c)\mathbf{n}]\cdot\mathbf{r}} + B_2e^{(2\pi i/h)[\mathbf{p}-(h\nu/c)\mathbf{n}]\cdot\mathbf{r}+i\delta}, \\ w_- &= C_1e^{(2\pi i/h)[\mathbf{p}-(h\nu/c)\mathbf{n})\cdot\mathbf{r}} + C_2e^{(2\pi i/h)(\mathbf{p}+(h\nu/c)\mathbf{n}]\cdot\mathbf{r}-i\delta}. \end{aligned} \quad (38)$$

Since the relation (24) holds between the p's and E this gives for the values of the constants

$$\begin{aligned} B_1 &= -C_1 = \frac{e}{h\nu}\frac{\left(\mathbf{p}\cdot\mathbf{a}' - \frac{E}{c}a'\right)}{\left(\mathbf{p}\cdot\mathbf{n} - \frac{E}{c}\right)}, \\ C_2 &= -B_2 = \frac{e}{h\nu}\frac{\left(\mathbf{p}\cdot\mathbf{a}' - \frac{E}{c}a'\right)}{\left(\mathbf{p}\cdot\mathbf{n} + \frac{E}{c}\right)}. \end{aligned} \quad (39)$$

The solution of the perturbation equation is consequently

$$\begin{aligned} f = \frac{e}{h\nu}\frac{\left(\mathbf{p}\cdot\mathbf{a}' - \frac{E}{c}a'\right)}{\left(\mathbf{p}\cdot\mathbf{n} - \frac{E}{e}\right)}&\left[e^{(2\pi i/h)\{[\mathbf{p}+(h\nu/c)\mathbf{n}]\cdot\mathbf{r}-(E+h\nu)t\}}\right. \\ &\left. - e^{(2\pi i/h)\{[\mathbf{p}-(h\nu/c)\mathbf{n}]\cdot\mathbf{r}-(E-h\nu)t\}}\right] \\ + \frac{e}{h\nu}\frac{\left(\mathbf{p}\cdot\mathbf{a}' - \frac{E}{c}a'\right)}{\left(\mathbf{p}\cdot\mathbf{n} + \frac{E}{c}\right)}&\left[e^{(2\pi i/h)\{[\mathbf{p}+(h\nu/c)\mathbf{n}]\cdot\mathbf{r}-(E-h\nu)t\}-i\delta}\right. \\ &\left. - e^{(2\pi i/h)\{[\mathbf{p}-(h\nu/c)\mathbf{n}]\cdot\mathbf{r}-(E+h\nu)t\}+i\delta}\right]. \end{aligned} \quad (40)$$

The complete solution $\psi + \sigma f$ may then be written

$$e^{(2\pi i/h)(\mathbf{p}\cdot\mathbf{r}-Et)}\left[1+\sigma\frac{e}{h\nu}\cdot\frac{1}{2\sigma}\frac{\left(\mathbf{p}\cdot\mathbf{a}-\frac{E}{c}a\right)}{\left(\mathbf{p}\cdot\mathbf{n}-\frac{E}{c}\right)}\left\{e^{(2\pi i/h)[(h\nu/c)\mathbf{n}\cdot\mathbf{r}-h\nu t]}\right.\right.$$

$$\left.-e^{-(2\pi i/h)[(h\nu/c)\mathbf{n}\cdot\mathbf{r}-h\nu t]}\right\}+\sigma\frac{e}{h\nu}\cdot\frac{1}{2\sigma}\frac{\left(\mathbf{p}\cdot\mathbf{a}-\frac{E}{c}a\right)}{\left(\mathbf{p}\cdot\mathbf{n}+\frac{E}{c}\right)}$$

$$\left.\times\left\{e^{(2\pi i/h)[(h\nu/c)\mathbf{n}\cdot\mathbf{r}+h\nu t]-i\delta}-e^{-(2\pi i/h)[(h\nu/c)\mathbf{n}\cdot\mathbf{r}+h\nu t]+i\delta}\right\}\right]$$

$$=e^{(2\pi i/h)(\mathbf{p}\cdot\mathbf{r}-Et)}\left[1+\frac{e}{2\pi\nu}\frac{\left(\mathbf{p}\cdot\mathbf{a}-\frac{E}{c}a\right)}{\left(\mathbf{p}\cdot\mathbf{n}-\frac{E}{c}\right)}\frac{2\pi i}{h}\sin\frac{2\pi\nu}{c}(\mathbf{n}\cdot\mathbf{r}-ct)\right.$$

$$\left.-\frac{e}{2\pi\nu}\frac{\left(\mathbf{p}\cdot\mathbf{a}-\frac{E}{c}a\right)}{\left(\mathbf{p}\cdot\mathbf{n}+\frac{E}{c}\right)}\frac{2\pi i}{h}\sin\left\{\frac{2\pi\nu}{c}(-\mathbf{n}\cdot\mathbf{r}-ct)+\delta\right\}\right]. \quad (41)$$

For the sake of comparison solution (34) may be written

$$e^{(2\pi i/h)(\mathbf{p}\cdot\mathbf{r}-Et)}\left[1+\frac{2\pi i}{h}A_1\sin\theta_1+\frac{2\pi i}{h}A_2\sin\theta_2\right]. \quad (34a)$$

since the A's are small, and this, in view of the known values of A_1 and A_2 (31 and 32) is seen to be identical with (41). This establishes the essential identity of the two processes worked out by Gordon and Klein, the erroneous sign appearing in Klein's equation (88) being obviously an oversight.

This simple solution corresponds to but one of the many possible solutions of (20); the appropriate solution for a given problem is to be developed as an infinite sum in terms of these separate simple solutions, each of which before summation is multiplied by a normalizing coefficient $C(p)$

and by an arbitrary function $z(p)$. This arbitrary function $z(p)$ is determined from the initial conditions of the problem; i.e. at $t = 0$, the coefficients $z(p)$ must be adjusted to fit the given conditions. In this case, inasmuch as the electron may have any momentum whatsoever, adding all the corresponding values of ψ must throw the sum into the form of an integral

$$\psi = \int_{-\infty}^{\infty} e^{(2\pi i/h)(\mathbf{p}\cdot\mathbf{r}-Et)} e^{(2\pi i/h)(A_1 \sin\theta_1 + A_2 \sin\theta_2)} C(p) z(p) dp, \quad (42)$$

where $dp = dp_x dp_y dp_z$.

3. *Derivation of the current density expression.* According to classical theory the vector and scalar potentials of the emitted radiation are to be expressed as the retarded potentials of the current and charge densities. Hence our next problem is to derive expressions for these densities. Following Gordon and Klein we may express these densities in terms of the ψ functions according to the following relations:

$$\begin{aligned} \rho\mathbf{u} &= \frac{e}{2m_0}\frac{h}{2\pi i}\left(\bar{\psi}\,\mathrm{grad}\,\psi - \psi\,\mathrm{grad}\,\bar{\psi} - \frac{4\pi i}{h}\frac{e}{c}\mathbf{U}\psi\bar{\psi}\right) \\ \rho &= -\frac{e}{2m_0c^2}\frac{h}{2\pi i}\left(\bar{\psi}\frac{\partial\psi}{\partial t} - \psi\frac{\partial\bar{\psi}}{\partial t} + \frac{4\pi i}{h}e\varphi\psi\bar{\psi}\right). \end{aligned} \quad (43)$$

These can be shown to qualify as current and charge densities because they satisfy the equation of continuity. We shall content ourselves with finding the expression for the current density, since the same relations as regards frequency and momentum conditions would grow out of the charge density and the resulting scalar potential. The evaluation of the separate terms of $\rho\mathbf{u}$ involves the multiplication of such integrals as $\bar{\psi}$ and grad ψ, and $\rho\mathbf{u}$ becomes

$$\begin{aligned} \rho\mathbf{u} = \frac{e}{2m_0}\int\int\Big[&(\mathbf{p}+\mathbf{p}') + \left\{\frac{2\pi\nu}{c}\mathbf{n}(A_1 + A_1') - \frac{2e}{c}\mathbf{a}\right\}\cos\theta_1 \\ &+ \left\{\frac{2\pi\nu}{c}\mathbf{n}(-A_2 - A_2') - \frac{2e}{c}\mathbf{a}\right\}\cos\theta_2\Big] \\ &\times e^{(2\pi i/h)\delta W} z(p) z(p') C(p) C(p') dp\, dp', \end{aligned} \quad (44)$$

where

$$\begin{aligned} \delta W = -(E - E')t + (\mathbf{p} - \mathbf{p}')\cdot\mathbf{r} + (A_1 - A_1')\sin\theta_1 \\ + (A_2 - A_2')\sin\theta_2, \end{aligned}$$

where p' is an arbitrary variable of integration, where E' is defined by the relation $(E'^2/c^2) - m_0^2c^2 = \mathbf{p}'^2$, and where the A''s are the same function of the p''s that the A's are of the p's. The range of integration of the p''s is from minus infinity to plus infinity as for the p's. The factor $e^{(2\pi i/h)\delta W}$ may be written

$$e^{(2\pi i/h)[(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}-(E-E')t]}\left[1+\frac{2\pi i}{h}(A_1-A_1')\sin\theta_1+\frac{2\pi i}{h}(A_2-A_2')\sin\theta_2\right], \quad (45)$$

since the terms in the A's are small; the next term in the Taylor's expansion would involve terms of the order of those which we are neglecting. We may use this value of $e^{(2\pi i/h)\delta W}$ in equation (44), giving

$$\rho\mathbf{u} = \frac{e}{2m_0}\int\int\left[(\mathbf{p}+\mathbf{p}')+\left\{\frac{2\pi\nu}{c}\mathbf{n}(A_1+A_1')-\frac{2e}{c}\mathbf{a}\right\}\cos\theta_1+\frac{2\pi i}{h}(\mathbf{p}+\mathbf{p}')(A_1-A_1')\sin\theta_1+\left\{\frac{2\pi\nu}{c}\mathbf{n}(-A_2-A_2')-\frac{2e}{c}\mathbf{a}\right\}\cos\theta_2+\frac{2\pi i}{h}(\mathbf{p}+\mathbf{p}')(A_2-A_2')\sin\theta_2\right] e^{(2\pi i/h)[(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}-(E-E')t]}z(p)z(p')C(p)C(p')dpdp'. \quad (46)$$

This expression may be simplified by expressing the cosine as $\frac{1}{2}(e^{i\theta}+e^{-i\theta})$ and $i\sin\theta$ as $\frac{1}{2}(e^{i\theta}-e^{-i\theta})$. This will give rise to terms with coefficients of the t exponent $-(E-E'+h\nu)$ and $-(E-E'-h\nu)$, and upon interchanging p and p' (which values will occur since we are integrating both p and p' throughout all possible values) to terms in $(E-E'-h\nu)$ and $(E-E'+h\nu)$. The second and third of these expressions correspond to an increase in the frequency of the emitted radiation which at least in the realm of X-rays has not been experimentally shown; hence we will neglect the terms involving these exponents. Terms of a similar character have, however, been observed in the visible region in the Raman effect. The terms involving the two remaining exponents of t may both be considered by using the real part only of the integral

$$\rho\mathbf{u} = \int\int[(\mathbf{p}+\mathbf{p}') + \mathbf{T}_1 e^{i\theta_1} + \mathbf{T}_2 e^{i\theta_2}] e^{(2\pi i/h)[(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}-(E-E')t]} z(p)z(p')C(p)C(p')dpdp', \quad (47)$$

where

$$\mathbf{T}_1 = \left\{\frac{2\pi\nu}{c}\mathbf{n}(A_1+A_1') - \frac{2e}{c}\mathbf{a}\right\} + \frac{2\pi}{h}(\mathbf{p}+\mathbf{p}')(A_1-A_1'),$$
$$\mathbf{T}_2 = \left\{\frac{2\pi\nu}{c}\mathbf{n}(-A_2-A_2') - \frac{2e}{c}\mathbf{a}\right\} + \frac{2\pi}{h}(\mathbf{p}+\mathbf{p}')(A_2-A_2'). \quad (47a)$$

4. *Derivation and significance of the vector potential of the emitted radiation.* This value of the current density must now be inserted in the usual expression for the retarded potential to give the vector potential of the emitted radiation,

$$\mathbf{U}' = \frac{1}{c}\int\frac{[\rho\mathbf{u}]}{R}d\tau, \quad (48)$$

where $d\tau = dx\,dy\,dz$. This gives the sum of three nine-fold integrals

$$\mathbf{U}' = \mathbf{U}_0' + \mathbf{U}_1' + \mathbf{U}_2', \quad (49)$$

where

$$\mathbf{U}_0' = \frac{e}{2m_0Rc}\int\int\int(\mathbf{p}+\mathbf{p}')e^{(2\pi i/h)\{(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}-(E-E')[t-(R/c)+(\xi\cdot\mathbf{r}/c)\}} \times z(p)z(p')C(p)C(p')dp\,dp'd\tau, \quad (49a)$$

$$\mathbf{U}_1' = \frac{e}{2m_0Rc}\int\int\int\mathbf{T}_1 e^{(2\pi i/h)\{(h\nu/c)\mathbf{n}\cdot\mathbf{r}-h\nu[t-(R/c)+(\xi\cdot\mathbf{r}/c)]\}} \times e^{(2\pi i/h)\{(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}-(E-E')[t-(R/c)+(\xi\cdot\mathbf{r}/c)]\}} \times z(p)z(p')C(p)C(p')dp\,dp'd\tau, \quad (49b)$$

$$\mathbf{U}_2' = \frac{e}{2m_0Rc}\int\int\int\mathbf{T}_2 e^{(2\pi i/h)\{-(h\nu/c)\mathbf{n}\cdot\mathbf{r}-h\nu[t-(R/c)+(\xi\cdot\mathbf{r}/c)]\}+i\delta} \times e^{(2\pi i/h)\{(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}-(E-E')[t-(R/c)+(\xi\cdot\mathbf{r}/c)]\}} \times z(p)z(p')C(p)C(p')dp\,dp'd\tau, \quad (49c)$$

and where for t has been substituted its retarded value $\left(t-\frac{R}{c}+\frac{\boldsymbol{\xi}\cdot\mathbf{r}}{c}\right)$. R is here the distance from the point of observation to the center of the electron and $\boldsymbol{\xi}$ is the unit vector in the direction of observation.

The integration of these nine-fold integrals with respect to six of the variables, p' and τ, may be easily effected by throwing that part of the integral into the form of a Fourier integral, as is frequently done in Schroedinger's theory. The characteristics of such a Fourier integral are indicated by the following relation:

$$h^3F(P, P) = \int\int F(P, P')e^{(2\pi i/h)(\mathbf{P}-\mathbf{P}')\cdot\mathbf{r}}dP'd\tau \qquad (50)$$

where $F(P, P')$ symbolizes some function of P_x, P_y, P_z, P'_x, P'_y, P'_z, and we see that the integrated function is exactly h^3 times the function that appears in the integrand multiplying the required exponential term but with $\mathbf{P}'$ replaced by $\mathbf{P}$.

Let us first apply this method of integration to $\mathbf{U}_0'$. That will require a change of variable such that

$$\mathbf{P} = \mathbf{p} - \frac{E}{c}\boldsymbol{\xi} \quad \text{and} \quad \mathbf{P}' = \mathbf{p}' - \frac{E'}{c}\boldsymbol{\xi}.$$

The function $F(P, P')$ is

$$(\mathbf{p} + \mathbf{p}')e^{(2\pi i/h)(E-E')[t-(R/c)]}z(p)z(p')C(p)C(p')JJ',$$

where J is the Jacobian of P and J' of P'. The resulting integral is

$$\mathbf{U}_0' = \frac{eh^3}{2m_0Rc}\int [e^{(2\pi i/h)(E-E')[t-(R/c)]} \times (\mathbf{p} + \mathbf{p}')JJ'z(p)z(p')C(p)C(p')]_P dP, \qquad (51)$$

where the subscript P indicates that all the p's involved are to be expressed as functions of the P's and the p''s of the P''s and then $\mathbf{P}'$ is to be put equal to $\mathbf{P}$, making the resulting expression a function of P alone. Putting $\mathbf{P}'$ equal to $\mathbf{P}$ gives us the relation

$$\mathbf{p} - \mathbf{p}' - \frac{E - E'}{c}\boldsymbol{\xi} = 0 \qquad (52)$$

and that, since $\boldsymbol{\xi}$ is a unit vector, may be expressed as

$$(\mathbf{p} - \mathbf{p}')^2 = \frac{(E - E')^2}{c^2}. \qquad (53)$$

This condition leads necessarily, because of (24), to the relations $E = E'$ and $\mathbf{p} = \mathbf{p}'$. The t exponent is thus made to vanish causing this part of the vector potential to be independent of the time. Consequently $\mathbf{U}_0'$ will emit no radiation and that fact alone causes us immediately to lose interest in it.

The second and third integrals, $\mathbf{U}_1'$ and $\mathbf{U}_2'$, will however give a contribution to the emitted electromagnetic radiation. Let us consider first $\mathbf{U}_1'$. The exponent here may be written as

$$-\frac{2\pi i}{h}(E - E' + h\nu)\left(t - \frac{R}{c}\right) + \frac{2\pi i}{h}\left\{(\mathbf{p} - \mathbf{p}')\cdot\mathbf{r} - \frac{E - E' + h\nu}{c}\boldsymbol{\xi}_1\cdot\mathbf{r} + \frac{h\nu}{c}\mathbf{n}\cdot\mathbf{r}\right\}. \quad (54)$$

We shall write $\boldsymbol{\xi}_1$ instead of $\boldsymbol{\xi}$ for the direction of observation merely to emphasize the fact that it belongs to this part of the integral $\mathbf{U}_1'$. The first parenthesis depending on t and the distance R is independent of r, and hence forms part of the function $F(P, P')$ in the Fourier integral (50); the second parenthesis must be expressed as $(\mathbf{P} - \mathbf{P}')\cdot\mathbf{r}$. This may be accomplished by changing the variables to

$$\begin{aligned}\mathbf{P} &= \mathbf{p} - \frac{E + h\nu}{c}\boldsymbol{\xi}_1 + \frac{h\nu}{c}\mathbf{n},\\ \mathbf{P}' &= \mathbf{p}' - \frac{E'}{c}\boldsymbol{\xi}_1.\end{aligned} \quad (55)$$

To integrate with respect to these new variables one must multiply the integrand by the Jacobian of P, J, and by the Jacobian of P', J'. These factors will both form part of the function $F(P, P')$. Equations (55) together with the relation (24) enable us to express the p's and E as functions of the P's, and also the p''s and E' as functions the P''s. Replacing $\mathbf{P}'$ by $\mathbf{P}$ is equivalent to replacing $\mathbf{p}' - (E'/c)\boldsymbol{\xi}_1$ by $\mathbf{p} - \boldsymbol{\xi}_1(E + h\nu)/c + (h\nu/c)\mathbf{n}$, i.e. to replacing $\mathbf{p} - \mathbf{p}' - \boldsymbol{\xi}_1(E - E' + h\nu)/c + (h\nu/c)\mathbf{n}$ by zero. In other words, remembering that $|\mathbf{p}'|$ is equal to $\sqrt{(E'^2/c^2) - m_0^2c^2}$, we see that replacing $\mathbf{P}'$ by $\mathbf{P}$ is equivalent to replacing E' by a quantity E_{1P} which is a solution of the equation

$$\mathbf{j}_1\sqrt{\frac{E_{1P}^2}{c^2}-m_0^2c^2}-\frac{E_{1P}}{c}\boldsymbol{\xi}_1=\mathbf{p}-\frac{E+h\nu}{c}\boldsymbol{\xi}_1+\frac{h\nu}{c}\mathbf{n}, \quad (56)$$

where $\mathbf{j}_1$ is a unit vector the direction of which is determined by this relation. We have then for the resulting expression for $\mathbf{U}_1'$

$$\mathbf{U}_1'=\frac{eh^3}{2m_0cR}\int e^{-(2\pi i/h)(E-E_{1P}+h\nu)[t-(R/c)]} \times[\mathbf{T}_1C(p)C(p')z(p)z(p')JJ']_P dP, \quad (57)$$

where the subscript P again indicates the same thing that it did in (51).

If we should now attempt to complete the integration of the vector potential (57) which we have caused to be a function of the initial electron momentum $\mathbf{p}$, and of ν, $\mathbf{n}$ and $\boldsymbol{\xi}$, we should be attempting to consider simultaneously all vector potentials which might exist and all emitted radiations which occur when an X-ray wave of a known frequency encounters electrons of all possible velocities. We shall consequently proceed as we do in the case of an atomic transition and pick out one term corresponding to a given transition or more precisely to one given initial velocity of the electron. To do this we must separate the integrand of (57) into two parts, one determining the intensity of the component waves and the other the number of terms in the infinite sum lying between p and $p+dp$. This requires us to give a more definite meaning to the series coefficients $z(p)$, for we must find for the new system of coördinates P a function $Z(P)$ which will have as nearly as possible the same characteristics as $z(p)$ in the old. We shall place upon our solution (42) for the initial state of the electron, in which λ is to be taken as zero, the condition that the integral of the expression for the density (43) using this value of ψ taken over all space shall give the electronic charge e. The total charge then takes the form of the integral

$$e=\frac{eh^3}{m_0c^2}\int EC^2(p)z^2(p)dp, \quad (58)$$

where the integration with respect to $d\tau$ and dp' has already

been carried out using the Fourier integral theorem. Before we can give a definite meaning to $z(p)$ we must evaluate $C(p)$, the normalization constant. For this we shall use the physical criterion of normalization stated by Klein which holds for the relativistic as well as for the non-relativistic case, that for each state the density integrated over all space must be equal to the electronic charge e, but since this integral is here divergent we shall in accord with the procedure adopted by Born [6] use the mean value of this integral over the integration range instead of its actual value. Thus we have for the simple solution $\psi = C_s e^{(2\pi i/h)(\mathbf{p}_s \cdot \mathbf{r} - E_s t)}$ the condition on C_s that

$$\lim_{\tau \to \infty} \frac{1}{\tau} \int C_s^2 \frac{eE_s}{m_0 c^2} d\tau = e. \tag{59}$$

In the limit this gives for C_s the value

$$C_s^2 = \frac{m_0 c^2}{E_s}. \tag{60}$$

For the non-relativistic case C_s is then unity and the average value of $\int \psi \bar{\psi} d\tau$ is also unity in accord with the usual condition for normalization. This value substituted in (58) gives the condition

$$1 = h^3 \int z^2(p) dp \tag{61}$$

which differs by the factor h^3 from that given by Gordon, a difference which is immaterial since it is a constant factor. Moreover, by comparing (59) which is the case for an electron in a single state with (58) which has been derived from ψ expressed as a sum of terms, the two are seen to be consistent if we arbitrarily take $h^3 z^2(p) dp$ as the number of terms involved between p and $p + dp$ which are to be taken in order to give the initial distribution of charge. In changing to another set of coördinates P, the number of terms to be taken in a corresponding range dP to give the same initial distribution must be the same and we can define a new function $Z(P)$ such that

$$z^2(p) dp = Z^2(P) dP, \tag{62}$$

[6] Born, M.: *ZS. f. Physik*, 38, 803, 1926.

so that if J is the Jacobian of the transformation, for which $J\,dP = dp$, we have

$$[z(p)]_P\sqrt{J} = Z(P). \tag{63}$$

We have then for $\mathbf{U}_1'$, on substituting the value of $C(p)$ already found, the expression

$$\mathbf{U}_1' = \frac{ec}{2R}h^3\int e^{-(2\pi i/h)(E-E_{1P}+h\nu)[t-(R/c)]}\frac{\mathbf{T}_{1P}}{\sqrt{EE_{1P}}} \times\left[Z(P)Z'(P)\sqrt{\frac{J'}{J}}\right]_P dp, \tag{57a}$$

in which it is to be observed we have changed the variable of integration back to p. $Z'(P)$ here is defined analogously to $Z(P)$ as $[z(p)]_P\sqrt{J'}$ and thus obviously is a different function from $Z(P)$. If we now work out the vector potential corresponding to the transition between two states of energy designated by E and by E_{1P}, we find

$$\mathbf{U}_1' = \frac{ec}{2R}\frac{\mathbf{T}_1}{\sqrt{EE_{1P}}}e^{-(2\pi i/h)(E-E_{1P}+h\nu)[t-(R/c)]}. \tag{64}$$

We can now set up a correspondence between this expression and the expression represented in equation (57a) by saying that the integral is the equivalent of having $h^3Z(P)Z'(P)\sqrt{J'/J}$ contributions per unit range of p in the vector potential of amount equal to that given by equation (64), each of these terms representing a transition from the energy state E to the energy state E_{1P}.

It may be noted that we here differ slightly from the procedure adopted by Gordon, who retains the variable P in the expression (57a), so that the states are symbolized by P instead of by p, and the number of them per unit range of P is taken as $h^3Z(P)Z'(P)$. This results in the assignment of an amplitude $\frac{ec}{2R}\frac{\mathbf{T}_{1P}\sqrt{JJ'}}{\sqrt{EE_{1P}}}e^{-2\pi i\nu_{1P}[t-(R/c)]}$ to each state, a form of expression differing from that we have chosen by the factor $\sqrt{JJ'}$. It would certainly seem that the number of states should be specified in terms of p rather than of P.

However, this operation of procedure only plays a minor part. For example, it does not affect the fact that the intensity of the emitted radiation is proportional to the cube of the frequency.

Carrying out a similar process for the integral $\mathbf{U}_2'$, we get for the final result

$$\mathbf{U}_2' = \frac{ec}{2R} e^{i\delta} \frac{\mathbf{T}_{2P}}{\sqrt{EE_{2P}}} e^{-(2\pi i/h)(E-E_{2P}+h\nu)[t-(R/c)]}. \tag{65}$$

The replacement of $\mathbf{P}'$ by $\mathbf{P}$ indicated by the subscript P in (65) is the equivalent of replacing

$$\mathbf{p}' - \frac{E'}{c}\boldsymbol{\xi}_2 \quad \text{by} \quad \mathbf{p} - \frac{E+h\nu}{c}\boldsymbol{\xi}_2 - \frac{h\nu}{c}\mathbf{n}. \tag{66}$$

The phase angle δ merely introduces a constant factor $e^{2\pi i/k}(k \geqq 1)$ which has unity as its maximum value. For certain values of δ this constant will have negative values. It thus affects the intensity and phase of the electromagnetic wave emitted due to this part of the integral, but the frequency in which we are particularly interested is not affected.

It remains for us to correlate the constants appearing in this solution with the motion of the recoil electron. Obviously the coefficient of the exponential in t determines the frequency emitted and the rest of the function determines the intensity of the radiation in the given direction. Hence we will define ν_{1P}, the frequency of the emitted radiation, by the relation

$$h\nu_{1P} = E - E_{1P} + h\nu \tag{67}$$

restricting ourselves for the time being to the consideration of $\mathbf{U}_1'$.[7] The terms $h\nu$ and $h\nu_{1P}$ represent the energy of the

[7] The term is supposed to govern the rate of radiation of electromagnetic energy in the following steps. We are first to carry out a calculation analogous to the above for the scalar potential. It is then necessary to calculate the electric and magnetic fields in terms of these potentials, and to calculate the electromagnetic Poynting flux per unit solid angle with origin at the point designated by R. This quantity will of course be proportional to the square of the amplitude referred to. We are to divide this rate of radiation of electromagnetic energy by $h\nu_{1P}$, and the result is to be taken as proportional to the probability of the tran-

incident and emitted X-rays and E was shown to be the initial energy of the electron. Hence, if we are to provide for a situation in which conservation of energy is to hold, E_{1P} must be associated with the energy of the recoil electron. In this connection it is to be noted that there is no other quantity in our theory which has a prior claim to this title. It is also appropriate to associate with E_{1P} a quantity $\mathbf{p}_{1P}$, defined by $|\mathbf{p}_{1P}| = \sqrt{(E_{1P}^2/c^2) - m_0^2c^2}$, which is the momentum of the recoil electron. This quantity E_{1P} is given by equation (56) which since it represents three equations serves to determine both E_{1P} and also j_{1x}, j_{1y}, j_{1z} in view of the relation $j_{1x}^2 + j_{1y}^2 + j_{1z}^2 = 1$. The determination of E_{1P} of course fixes for us the magnitude of the recoil momentum of the electron and of $\mathbf{j}_1$ the direction of emission of this electron in terms of the foregoing hypotheses. Thus (56) may be written, in view of the definitional relation (67), as

$$\mathbf{p}_{1P} + \frac{h\nu_{1P}}{c}\boldsymbol{\xi}_1 = \mathbf{p} + \frac{h\nu}{c}\mathbf{n}. \tag{68}$$

(56) now appears as identical with the Compton-Debye condition for the conservation of momentum for the system formed by the electron and the quantum, if we christen $(h\nu/c)\mathbf{n}$ and $(h\nu_{1P}/c)\boldsymbol{\xi}_1$ the momenta of the incident and emitted quanta as is the custom. Looked at from another point of view, this procedure gives us the right so to christen them. A similar correlation may be made for the condition (66) accompanying the integration of $\mathbf{U}_2'$, which leads to the final relation

$$\mathbf{p}_{2P} + \frac{h\nu_{2P}}{c}\boldsymbol{\xi}_2 = \mathbf{p} - \frac{h\nu}{c}\mathbf{n}, \tag{69}$$

where $\mathbf{p}_{2P}$ is the momentum of the recoil electron in that case,

sition associated with the amplitude in question. In this connection it is to be noted that associated with any one transition there is radiation of electromagnetic energy in one and only one direction, the situation being analogous to what we have in the case of reflection of an electromagnetic wave from a grating. The method of specifying the probability above outlined results in the conclusion that when the probability of the transition is re-multiplied by $h\nu_{1P}$ it yields for a solid angle a rate of radiation of energy equal to that calculable on classical electromagnetic theory.

and ν_{2P} is defined by

$$h\nu_{2P} = E - E_{2P} + h\nu, \tag{70}$$

E_{2P} being the energy of the recoil electron. There is, however, no necessity that either the direction of observation nor the momentum of the recoil electron should be the same in the two integrals and hence I have used a different notation in expression $\mathbf{U}_2'$.

Conclusion. To interpret these results, the first obvious thing is that the vector potential of the emitted radiation is the sum of two separate potentials due to the separate effects of the two component waves. This was not immediately obvious without carrying through the computation. The conditions for each of these, (68) and (69), and the definitions, (67) and (70), are the usual Compton-Debye equations for such encounters, for the two waves one moving diametrically opposite to the other so that $(-\mathbf{n})$ appears in one equation and $(+\mathbf{n})$ in the other. There is a certain probability governed by the functions $\mathbf{T}_1$ and $\mathbf{T}_2$ that the electron will be affected by one or the other of the component waves so that its motion and the emitted radiation will then be determined by these energy-momentum conditions. According to the old electromagnetic theory, the picture presented to our minds would be one of an electromagnetic field composed of two oppositely directed trains of waves operating as a whole on the electron. There would be nothing to differentiate the action of the two separate trains of waves; whenever one of them did anything the other would do something. For even the faint potentialities inherent in the fact that a unidirectional wave would possess a Poynting flux would now be lost to this combined wave, at any rate in positions of zero Poynting flux. Now, however, the analysis has of its own accord seen a reason for splitting this stationary wave up into two oppositely directed parts. It has done this by realizing that at any rate to an approximation and only to an approximation, the solution of the whole equation leads to an expression for the vector potential for the emitted radiation consisting of the sum of two parts, one depending on one of these waves and the other depending on the other, and we may say that it would

not be content with any other method of splitting up this composite wave which we might choose for ourselves. It then results from the interpretation of our results in terms of probability considerations that each of these two trains of waves are able to act in the sense indicated, that is, that the scattering of the two component waves is to be considered quite independently one of the other.

If, for two electrons moving in the same direction with the same velocity, it should happen that each responded to one of the component running waves and scattered an X-ray quantum, one could compare the radiation scattered by the two electrons in a given direction of observation. Assuming that the radiation is scattered by both electrons in the same direction gives us the condition $\xi_1 = \xi_2$. Equations (68) and (69) would determine $\mathbf{p}_{1P}$ and $\mathbf{p}_{2P}$ since they each furnish three equations between the three unknown components of the recoil momentum. ν_{1P} and ν_{2P} are known functions of $\mathbf{p}$, $\mathbf{p}_{1P}$ and $\mathbf{p}_{2P}$, and so would not prevent this determination. Thus the momenta of the recoil electrons would be different both as regards direction and magnitude. The frequency being a function of the recoil momentum as well as of the initial momentum of the electron would also be different. This means that an observer would with one setting of his instrument find two scattered lines both of frequencies differing from that of the incident X-ray. Of course, there is nothing remarkable in this, as will be immediately evident to the experimentalist who would naturally expect that if he sent a beam of X-rays in one direction he would get a line in one position and that if he sent it in the opposite direction he would get a line in another position; he would therefore not be very much surprised if he sent the two beams together to get these two lines.

We may discuss from a slightly different point of view the fact that giving the velocity of the electron before impact in magnitude and direction, the frequency of the incident X-ray, and the directions of both the incident and emitted radiation determines uniquely the frequency of the emitted quantum and the vector momentum of the recoil electron. According to Schroedinger's [8] conception of the reflection of an electro-

[8] Schroedinger: *Ann. d. Physik*, 82, 257, 1927.

magnetic wave from the charge density wave, formed by the sum of the ψ waves associated with the initial and recoil electrons, as taking place according to Bragg's law for the first order, for a given incident X-ray wave and a specified direction of the scattered ray, *only* one charge density wave will qualify as a grating to be used for reflection. If the initial motion of the electron is given, there is consequently but one value of the velocity of the recoil electron which will produce this grating. In other words, for one assigned initial state there is one and only one other final state of the electron which will produce a grating appropriate to the reflection of our X-ray in the given assigned direction. The change in frequency can be computed as a Doppler effect due to reflection from this moving mirror the velocity of which is known. In this way we can explain more completely the thought underlying the analysis carried out in this paper. In forming our expression for $\rho\mathbf{u}$ we summed over all possible values of $\mathbf{p}$ and $\mathbf{p}'$, which may be identified with the momentum of the electron before and after impact—that is, physically we built a double infinity of these Schroedinger gratings. At the end we picked out one value of $\mathbf{p}$, the initial momentum of the electron, and that corresponded to cutting down the number of our gratings to a single infinity, but even with this restriction since $\mathbf{p}'$ could have any value we had a set of gratings with all possible velocities, directions and wave-lengths. However, owing to the mutual interference of the radiation from the various parts of these gratings it results that any one of them sends electromagnetic radiation off in one and only one direction. Moreover, if we specify the frequency of an X-ray wave and its incident and reflected directions there would be but one of these many gratings which could reflect for this case. The radiation travelling in the opposite direction would pick out another grating for its purpose. As the direction of observation is changed, two different gratings would be chosen automatically. Hence the infinite set of gratings would include all the individual gratings which are necessary to tell the story for all possible directions of observation or, for that matter, for all directions and frequencies of the incident radiation. The selective character of this grating reflection is indicated in the analysis in that the

integrals have no value unless their space exponents vanish and the condition that they vanish is equivalent to the conservation of energy and of momentum which is the same condition that Schroedinger derives from his grating analogy. In fact the final expression for $\mathbf{U}'$, the vector potential, in integral form represents exactly what we would have if we considered a number of gratings of the above kind, each one operative in respect of an initial beam of X-rays for one and only one direction of reflection.[9]

In conclusion it is a pleasure to thank Prof. W. F. G. Swann for the inspiration and help which he has so generously given during long discussions of the subject considered in this paper.

[9] It is satisfying to note that Bothe (*ZS. f. Physik*, 41, 332, 1927) has shown *experimentally* that the component waves of a standing X-ray wave are scattered separately, thus verifying this theory.

Denicotinized Tobacco. Several processes may be used in the preparation of denicotinized tobacco. Nicotine may be removed by the use of suitable solvents; or the tobacco may be subjected to a resweating either by treatment with superheated steam or by heating in a vacuum chamber. E. M. BAILEY, O. L. NOLAN, and W. T. MATHIS (*Conn. Agric. Exp. Station Bull.* 295, 338–351, 1928) have analyzed 17 samples of denicotinized tobacco as found on the market in cigars, cigarettes, and smoking tobacco. For comparison they also analyzed 18 samples of ordinary tobacco as found on the market, and collected from the literature 40 additional analyses of ordinary tobacco as found in cigars, cigarettes, and smoking and chewing tobacco. The total nicotine content, calculated on the moisture-free basis, ranged between 0.47 and 3.63 with an average of 1.96 per cent. in the 58 analyses of ordinary tobacco, and between 0.74 and 2.73 with an average of 1.41 per cent. in the 17 analyses of denicotinized tobacco. When tobaccos of corresponding leaf types were compared, the denicotinized products, as a rule, contained but little less nicotine than the ordinary products. While considerable reductions in nicotine content were indicated in certain instances, yet it was not difficult to find brands of ordinary tobacco with a nicotine content not greatly in excess of that present in the most thoroughly processed of the denicotinized products. Nicotine is an important, if not the chief, factor in the production of the satisfying effects derived from smoking. Use of denicotinized products may lead to increased indulgence; and the actual nicotine intake may equal or exceed that when ordinary tobacco is used. None of the denicotinized tobaccos analyzed were sufficiently low in nicotine content to warrant unrestricted indulgence by persons who suffer ill effects from this alkaloid.

Free nicotine occurs in tobacco, and is due, apparently, to the dissociation of the nicotine salts of organic acids. The harsh and irritating effects on smoking may be due to free nicotine. Denicotinized tobacco did not differ markedly from ordinary tobacco with respect to the free nicotine content. The distribution of the nitrogen as nitrate nitrogen and as ammoniacal nitrogen was practically identical in the two classes of tobacco.

J. S. H.

THE QUANTUM MECHANICS OF AN ELECTRON OR OTHER PARTICLE.

BY

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THE last three years have witnessed the performance of several physical experiments which bid fair to rival the most famous experiments of the last half century in revolutionary importance. Unlike most revolutionary experiments, however, their results have offered to the theoretical physicist a problem of explanation peculiarly baffling in character. The experiments deal with two entities which had, as it seemed, been demonstrated to be of radically different natures, namely, light and electrons, and in consequence of them we now see that the difference between these entities is not nearly so large as had been supposed. It is the purpose of this article to give an account in simple terms of the resulting changes in physical conceptions and of the manner in which the new phenomena and problems of a similar sort have been handled in the new theory of Quantum Mechanics.

The revolutionary experiments upon light were performed in 1925 by Bothe and Geiger in Germany and by Compton and Simon in this country. They are well described in A. H. Compton's article [1] in a recent number of this journal, also in his book on X-rays. The experiments actually deal with X-rays, but so many facts point to the conclusion that X-rays are merely light of extremely short wave-length that it seems quite safe to extend the general conclusions to ordinary light or to electromagnetic radiation of any other kind.

From these experiments we are led to the view that when a beam of X-rays passes through a piece of matter, nothing else happens at all during most of the time; the X-rays are not even scattered. Only now and then, here and there, does something cataclysmic happen in an atom, and, when

[1] A. H. Compton, JOUR. FRANK. INST., 205, 155, 1928.

it does, several things happen together: either a photo-electron is ejected from the atom and simultaneously a quantum of the incident X-rays disappears, or else both a slower recoil-electron is ejected and also a quantum or photon of X-ray energy starts off in a direction different from that of the incident X-rays to form part of the familiar scattered radiation. Such a scattered quantum does not, however, in accord with the wave-theory and like the waves of sound from a pistol-shot, spread out in all directions from the atom in the form of a spherical disturbance; instead of this, the entire quantum moves off in a fairly definite direction which can be calculated by simple rules from the direction in which the associated recoil-electron was ejected from the atom. The method of making this calculation is very similar to that employed in treating the collision of two billiard balls; it is just as if the incident X-ray photon had struck the electron, given up part of its energy to it and bounced off in a new direction, all in accord with the laws of elastic impact. It might seem at first sight as if we had been led back to the old corpuscular theory of light.

And yet there is every reason to believe that with these same X-rays interference experiments can be performed just as well as with ordinary light. For instance, we might allow a stream of these billiard-ball photons to fall upon a screen with a small hole in it and receive those which passed through the hole upon a second screen. We should then find that the streamlet which came through the hole did not continue quite straight ahead but spread sidewise somewhat, producing the phenomenon of diffraction. Now this effect we might easily explain, at least qualitatively, on the corpuscular hypothesis: perhaps those photons which pass near the edge of the hole are attracted by the material of the screen and are thereby deviated from their paths. But if we now experimented further and opened a second small hole near the first, thereby opening a path for a second streamlet to come through, we should find that the number of photons striking certain parts of the second screen was actually less than when only one hole was open, these parts forming the dark regions in the interference pattern. To judge from the results of experiments on very faint light, this effect

would occur even if only one photon per minute were coming through the holes; the blackening after many hours on a photographic plate placed behind the holes would still be less in certain places with both holes open than with only one open. The interference can therefore not be ascribed to interaction between different photons which are simultaneously present in the region.

Now such a result is practically impossible to explain on a corpuscular theory of the ordinary sort. It would seem that each photon, after passing through one hole, must understand how to lay down the entire interference pattern; it must be guided in its subsequent motion by the shape of all parts of the screen. Perhaps a set of laws could be set up which would guide the photons under all conditions to the right place,—possibly something like the combination of waves and corpuscles once tentatively suggested by Newton; but no one has yet elaborated any such explanation, and it seems to be the general conviction of theorists that, if it can be worked out at all, the result would be a theory at least inconveniently complicated. But before considering what the quantum theorists have done about the problem, let us turn to the other side of the experimental picture.

From 1900 to 1925 it was generally accepted that electrons, unlike light rays, were corpuscular in nature—little specks of electricity which could be thought of as moving inside a vacuum tube in a continuous manner along definite paths. There can hardly be any doubt that their energy and electric charge really are thus distributed discontinuously at least at the instant when they arrive at an electrode, for under certain circumstances they can be counted as they come (for instance by causing each electron to produce a visible "kick" of an electrometer fiber). And yet within the past two years [2] Davisson and Germer in America and Thompson in England have obtained definite interference effects with a beam of electrons! It now seems probable that even the simple interference experiment described above can be performed with such a beam, although it will be a difficult one because of the extremely short "wave-length" of electrons moving with appreciable energy.

[2] Cf., e.g., Davisson and Germer, *Nat. Acad. Sci., Proc.*, 14, 619, 1928; Thompson, *Nature*, 122, 279, 1928.

The chief difference in the mechanics of electrons and photons seems, in the light of these discoveries, to lie in the fact that electrons influence each other's motions (by means of their electromagnetic fields) whereas photons move independently of each other. With this exception the same type of law should be found to hold for both; and it must be something that is neither wave-theory nor corpuscular theory. Fortunately it happens that a new form of Quantum Mechanics has been emerging just during the past three years, arising out of the impasse into which the old quantum theory had gotten, and this new theory is equal to the task of describing the new facts adequately. The new theory is really not so much a fusion of the wave and the corpuscular theories as a remarkable combination of elements taken from both of them. It was initiated by Heisenberg in 1925, developed independently a little later in quite a different direction by deBroglie [3] and Schrödinger,[4] and finally completed a year ago for the purposes of "particle mechanics" by Heisenberg.[5] The theory is not yet complete, for the proper treatment of radiation processes is still partly obscure, as is also the mechanics of high-speed electrons. We shall accordingly limit the discussion in what follows to the case of particles—electrons or protons or atomic or molecular groups of these—moving with speeds small as compared to the speed of light. We shall also omit all mention of the earlier "matrix" form of the theory, which is mathematically equivalent to the other but is much more abstract and to many physicists repellent.

THE INDETERMINATION PRINCIPLE.

The great guiding idea of Heisenberg has been that in constructing our theories we ought to be very clear in our minds as to what quantities can really be made the object of physical observation; only observable quantities and such others as can be calculated from them by definite rules can be said to possess physical significance. The principle itself it as old as physics, of course; it is only Newton's "hypotheses non fingo" in a new dress. The novelty in Heisenberg's

[3] L. deBroglie, *Ann. de Physique*, 3, 22, 1925.
[4] E. Schrödinger, *Ann. d. Physik*, 79–81, several articles, 1926.
[5] W. Heisenberg, *Zeits. f. Phys.*, 43, 172, 1927.

position was his suggestion that the principle had not been sufficiently observed in the old quantum theory. Thus, atomic energy and spectral frequency are observable quantities, for, although they are pretty abstract conceptions and are far removed from the sights and sounds actually noted and recorded in a physical laboratory, yet they are calculable from the latter according to well-defined rules. On the other hand, orbital frequency and orbital size were never in the same class; from certain assumptions as to these, energy and spectral frequency could be calculated, but the process could not be reversed, as is well illustrated in the long and fruitless struggle on the part of the quantum theorists to discover the motions of the electrons in the helium atom.

Now the application of this idea to the concept, "motion of a particle," leads in the light of recent experimental discoveries to a remarkable conclusion. It appears that we must simply give up the idea of an electron as a particle occupying at successive instants a *continuous* series of successive positions; at least we must regard this idea as one which is purely imaginary and has no meaning in terms of physical observation. We must introduce into the conception of a moving particle a certain degree of haziness, a certain element of indeterminateness and mere probability entering fundamentally into the groundwork of natural processes.

It is not however, space-time itself that requires to be modified; space-time remains in the new theory as continuous and as infinitely divisible into elements as it ever was. It appears to be possible, for instance, at least in principle, to locate an electron at a certain exact point and at a certain exact moment, and the point and the moment may be anywhere on a continuous scale of spatial and temporal magnitudes. From the theoretical standpoint, the simplest method of so locating an electron is probably one similar to the method used by surveyors in finding the location of an inaccessible mountain-top, namely, to train two microscopes along converging lines upon a point in space and to contrive to "see" the electron in both instruments as it passes the point at a certain instant. Such an observation is scarcely possible as yet in practice, but this circumstance seems to inhere in difficulties of instrumental construction and manipulation

rather than in the nature of the laws governing natural processes. The method has the advantage of being pretty certainly capable of refinement to yield any desired degree of precision; we need only shorten sufficiently the wave-length of the light employed in order to attain the necessary resolving power in the microscopes. For instance, to attain a precision of one per cent. of an atomic diameter in the observed position, we should need to use light of about $\lambda = .01$ Ångstrom, i.e., soft γ-rays; but this seems to present no difficulty in principle, for it is now known that X-rays can be refracted like light and there is every reason to think that the same thing is true of all shorter waves. By employing first X-rays, then γ-rays, then cosmic rays and so on we might in principle attain any degree of precision in the observation of position.

Difficulties of another sort arise, however, when we pass from observations of position to the consideration of observations of motion. It is instructive to consider how we might attempt to observe the motion of an electron in a Bohr orbit.* We might take our cue from the astronomers and try to see it by means of our microscopes at successive points in the orbit. But then we should have to take account of one of the newly-discovered peculiarities of radiation, namely, the Compton effect; because of this, every time a pair of photons rebounds from the electron into our microscopes, the electron will receive an impulse which changes its motion. Thus we could never follow it round in a smooth Kepler ellipse; the observed points could at best only fall roughly on the ellipse. It seems a significant fact that this blurring of the observed path is at least enough to prevent the distinction by observation of any one quantum orbit from the next one. For instance, in a hydrogen-like atom, the radius of the nth circular orbit being $r = n^2h^2/4\pi^2me^2$, the difference in radius between the nth and $(n + 1)$st orbits is only $\Delta r = (2n + 1)h^2/4\pi^2me^2$. ($h$ = Planck's constant, m and e = mass and charge of the electron); to distinguish such a small difference in radial position we should have to employ light of $\lambda \leqq \Delta r/2$ (roughly speaking), which would impart a Compton impulse of magnitude

* Cf. Heisenberg, loc. cit.

$$\Delta p = \frac{h\nu}{c} = \frac{h}{\lambda} \geqq \frac{8\pi^2 me^2}{(2n+1)h},$$

more or less, and this would change the angular momentum by an amount of the order of $r\Delta p = 2n^2h/(2n+1)$; this expression has the value $2h/3$ for the first orbit, $8h/5$ for the second, and so on, whereas the difference in angular momentum between successive orbits of similar shape is only h.

It thus appears that we must cease to regard the Bohr orbits as possessing any physical reality. Similar considerations are believed to apply to the motion of any portion of matter that can be treated as a "particle." If this is correct, then orbital motion is a conception which has a relatively precise meaning only for phenomena on a certain scale of magnitude, such as planetary motion, where the effect even of billions of Compton kicks is negligible, whereas the conception becomes hazy when we turn to small-scale phenomena and loses all meaning if we go to the extreme of trying to distinguish adjacent "quantum" paths.

Closer analysis shows that the same difficulty inheres in any measurement of *velocity*. The fundamental mechanical definition of velocity is, of course, the distance between two positions occupied by the moving entity at two different times, divided by the difference in time. According to quantum mechanics, however, there will be an indefiniteness in the positions, if measured by the method described, of the order of λ, and there will be a Compton impulse upon the particle of the order of $h\nu/c$ or h/λ, so that at the instant when the position of the particle is determined its momentum (in any one direction) is undefined by an amount of order h/λ. The product of the degrees of indefiniteness in position and in momentum is thus of the order of h.

We have here an instance of an aspect of natural processes which evidently lies very near the root of those features which find their expression in the quantum theory. It is an instance of what has been called the Indetermination Principle of Heisenberg, the exact formulation of which is contained in equation (13) below. The principle is believed to hold quite generally, but its application often requires very painstaking

and clear reasoning and we shall refer for further discussion of it to the original literature.[5, 6, 7]

In consequence of the Indetermination Principle the fundamental concepts of the new mechanics differ radically from those of the old, and especially in two respects. In the first place, as we have seen, there is in the new theory a characteristic incompatibility between certain pairs of magnitudes when they are made objects of observation. This is a very important feature of the theory and neglect of it in dealing with special cases is very apt to lead to paradox. Thus, we have seen that when position is measured precisely, momentum must remain largely indefinite; whereas if we arrange a situation where momentum is accurately known (of which an example is given below), then the position is indefinite. In a similar way energy and motion are incompatible: when we follow a particle along a path, its energy remains somewhat indefinite; if on the other hand we select an atom in a certain quantum state characterized by a definite value of the energy, then, as we shall see presently, the process of selection has created a situation in which nothing referring to electronic motion appears in the observations.

In the second place, it follows from the Principle that an element of probability must necessarily enter into the statements that we make about physical systems and into our statements of physical laws. We can no longer say that a system of particles is at a time, t, definitely in a certain configuration and moving in a certain manner; we can only assign a probability function for the results that would be obtained if positions or momenta of the particles were to be measured with precision at that time. For example, when we have by suitable means isolated an atom in a definite quantum state, we cannot specify the position or motion of the electrons within it, we can only assign a probability function for the positions in which they would be found if they were to be located.

Accordingly it is to be expected that the fundamental laws of mechanics will have to do with this probability func-

[6] E. H. Kennard, *Phys. Rev.*, 31, 344, 1928.
[7] C. G. Darwin, *Roy. Soc. Proc.*, 117, 258, 1927.

tion or with some quantity closely related to it. This is the form in which the theory has been given by Schrödinger and Heisenberg, and we shall now turn to a more detailed consideration of this theory.

THE NEW PARTICLE DYNAMICS.

The principles of Quantum Mechanics are most easily grasped in direct comparison with those of classical theory.

In classical mechanics it was customary to write down each coördinate of a particle, Cartesian or generalized, as a function of the time, e.g. $q_n = q_n(t)$. These equations were tacitly interpreted as meaning that, if at a given time all of the particles were located experimentally, the values thus found for the coördinates would be the values of these functions for the given value of the time. The variation of the coördinates with the time was then assumed to occur in accordance with Newton's laws of motion, which might be dressed up in the form of the Hamiltonian equations, the Hamiltonian function being obtained from the forces that were assumed to act between the particles.

In the new mechanics, in accordance with what has been said above, we have in place of the coördinates as functions of the time a probability function for their values. The theory is most easily expressed, however, not in terms of the probability itself but in terms of what Pauli called the "probability amplitude"; if this is denoted by $\psi(q_1, q_2 \cdots q_n, t)$, or $\psi(q, t)$ for short, where $q_1 q_2 \cdots q_n$ are the generalized coördinates of the system, then the probability function is assumed to be $\psi\bar{\psi}$, where the bar over the second ψ means that the conjugate value is to be taken if ψ is a complex number. Thus the probability amplitude has, according to Heisenberg,[5] the fundamental significance that, if all of the n coördinates were to be determined with precision at a given moment, the chance of finding values for them lying between q_1 and $q_1 + dq_1$, q_2 and $q_2 + dq_2$, etc., would be

$$Pdq = \psi\bar{\psi}dq, \tag{1}$$

where dq stands for $dq_1 dq_2 \cdots dq_n$. This relationship between the amplitude and the probability is very similar to

the relation between amplitude and energy in any of the familiar types of wave-motion. It requires that always

$$\int \psi \bar{\psi} dq = 1, \tag{2}$$

the integral being extended over all physically admissible values of the coördinates.

In lieu of Newton's laws of motion or their equivalent it is then assumed as a fundamental law, following Schrödinger,[4] that ψ obeys the wave-equation*

$$\frac{h}{2\pi i}\frac{\partial \psi}{\partial t} = H\left(-\frac{h}{2\pi i}\frac{\partial}{\partial q}, q\right)\psi, \tag{3}$$

where $H\left(-\frac{h}{2\pi i}\frac{\partial}{\partial q}, q\right)$ stands for a differential operator formed from the Hamiltonian function by replacing in it each power of a momentum by the same power of $\left(-\frac{h}{2\pi i}\right)$ multiplied by a partial derivative of an order equal to the power, taken with respect to the corresponding variable. For example, in the case of one particle in a conservative field of force, if we employ Cartesian variables x, y, z and if p_x, p_y, p_z denote the corresponding components of momentum, then

$$H = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + V,$$

where V is the potential energy and m the mass of the particle, and (3) becomes

$$\frac{h}{2\pi i}\frac{\partial \psi}{\partial t} = -\frac{h^2}{8\pi^2 m}\Delta\psi + V\psi, \tag{4}$$

in which Δ stands for $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$. The determination of the Hamiltonian function is, of course, in all cases an important fundamental problem. Fixing this function corresponds in

* The train of ideas that led to this rather mysterious-looking postulate is to be found in the paper of deBroglie, loc. cit.

the new theory to fixing the constitution of the system in the old theory, from which the Hamiltonian was then deduced; its determination is thus a matter of hypothesis to be tested by agreement of deductions from it with experiment. Fortunately, carrying over the classical Hamiltonian for the Rutherford atom into the new theory has resulted in so many successes that the classical form of this function seems to be at least very nearly the correct one for atomic systems. The occurrence of the universal natural constant h in this equation constitutes, at least so far as mechanics is concerned, its fundamental point of entry into the theory of physical phenomena.

A knowledge of ψ covers in the new theory all that can be known about the state of the system. This may at first sight seem surprising in view of the fact that ψ contains only the coördinates whereas in classical theory one had to assign the values of the velocities or the momenta as well in order to describe the state of the system completely. The explanation lies in the fact that ψ, being complex, is, so to speak, a more powerful number than the real numbers of ordinary theory; if we write $\psi = Re^{i\theta}$, where R and θ are real functions of the q's and t, then $\psi\bar{\psi} = R^2$, so that the modulus R alone determines the probability while the argument θ is left free to express additional features of the situation corresponding to the velocities of the particles.

One of the most important types of problem in classical theory was one in which the initial positions and velocities of the particles were assumed to be known and their positions and velocities at future times were calculated. The corresponding problem in the new theory is obviously one in which the values of ψ at a certain initial time are assumed to be known; the future history of the system is then represented by that solution of the wave-equation which reduces to the given function of the coördinates at the initial moment. Such an initial value of ψ would be determined in the usual way by previous knowledge, by the conditions of an experiment, etc. The values assumed would be subject (at least in principle) to experimental check, only it is characteristic of the new theory that such a check would have to be made by a statistical method: we should have to repeat the experi-

ment or to reproduce the given situation a great many times and each time determine the position of the particles at the same moment relative to the time of beginning the repetition. The results would then vary from one repetition to the next, and by treating them statistically we should obtain the probability function $\psi\bar{\psi}$. A statistical study of this sort has actually been made in certain cases, as in Rutherford's observations upon the scattering of α-particles. From these observations we should then know the absolute value of ψ for all values of the coördinates; by repeating the determination at a slightly different time and introducing the assumption that ψ obeys the wave-equation, we could then arrive at the values of ψ itself, but we shall not discuss the mathematical method of doing this.

Such statistical studies are, however, comparatively rare in practice; usually an experiment is so arranged that the inevitable statistical variations are too small to be of importance, and experimental conditions are usually known in gross terms, such as size of an electrode or applied potential. The process of passing rigorously from such conditions to the complete value of ψ, which must contain the coördinates of all of the electrons or atoms forming the material under observation, is very complicated and has been replaced in all applications of the theory to date by a shrewd surmise as to the form which the result must take, to be checked by agreement of the calculated consequences with experiment; illustrations of this will be given below in the discussions of electrons having definite speeds and of atomic energy levels.

THE CLASSICAL THEORY AS AN APPROXIMATION.

Any new theory of mechanics must, of course, pass into the classical theory under those conditions where the classical theory constitutes a well-established first approximation to the truth; otherwise it could not be correct. The simplest connection of Quantum Mechanics with classical theory lies in those cases in which a particle is confined within a certain region of space, ψ being zero (or practically so) outside of that region. Such a distribution of ψ has been called a "wave-packet" (Schrödinger) or "probability packet" (Heisenberg). As time passes, such a packet will move about in accordance

with the wave-equation. This motion can often be resolved more or less into a translatory motion of the packet as a whole and a change in size; if the change in size is small relatively to the distance covered in the translatory motion, then the packet represents a particle whose position is definite to a certain extent and which can with a certain degree of definiteness be said to be moving with a certain velocity. If we suppose in addition that the residual indefiniteness in these magnitudes is small as compared to the scale of magnitude upon which the phenomena are being viewed, then we have the case in which the classical theory is approximately valid.

There are even some cases in which the translatory motion of a packet is exactly the classical motion of the particle itself; these are, in fact, the simplest cases of all, the necessary condition being that the space derivatives of the force shall be either zero or at least constant. Thus one finds [8,7] by solving (3) or (4) that the packet representing a particle in a uniform gravitational field "falls" with the classical acceleration, the packet for an electron in a uniform magnetic field moves round in a circle at the classical rate, and in a harmonic field the packet vibrates back and forth with the classical frequency. It is only in more complicated fields, like the Coulomb field around a nucleus, that novel results are obtained. In the case of a harmonic field the amplitude of the "center" of the packet may even have any value, there being no trace of quantisation in its motion. One cannot, however, quite assert that the particle moves in violation of the old quantum rule, for the size of the packet is always necessarily great enough to obliterate the difference between the motions in a successive pair of the old quantum states. Quantisation is a phenomenon, not of motion, but of energy or angular momentum, as we shall see below.

Usually, however, a packet not only moves but changes "size" as well; in the last two cases just mentioned, the size merely pulsates, but in the cases of free motion and of a uniform gravitational field the packet continually enlarges and, if it is very small to begin with, it spreads very rapidly. The latter behavior is intimately connected with the In-

[8] E. H. Kennard, *Zeits. f. Phys.*, 44, 326, 1927.

determination Principle: if the packet is initially small, its position is then fixed rather closely and there must be a correspondingly large indefiniteness in its velocity, which requires in turn a rapidly increasing indefiniteness in its subsequent positions and so a rapid enlargement of the packet with time. It is an interesting fact that in all of the simple cases just mentioned correct results are obtained also if we assume that the particle moves under classical laws but that its initial position and velocity are not precisely known, only a probability curve being assignable for each. In the case of other force fields, however, such as the Coulomb field, an interpretation of this kind is almost certainly impossible (no rigorous proof of this fact is known to the writer).

A ONE-DIMENSIONAL EXAMPLE.

A better idea of the theory can perhaps be got by considering a concrete example. Let us take the case of a particle moving in one direction in a uniform gravitational field. Its wave-equation can be written (cf. (4)).

$$\frac{h}{2\pi i}\frac{\partial\psi}{\partial t} = -\frac{h^2}{8\pi^2 m}\frac{\partial^2\psi}{\partial x^2} - mgx\psi, \tag{5}$$

and a possible packet is given by:

$$\psi = \frac{C}{\sqrt{1 + \frac{ht}{2\pi i m x_i^2}}} \exp\left[-\frac{\left(x - x_m - \frac{\xi_m}{m}t - \frac{1}{2}gt^2\right)^2}{2\left(x_i^2 + \frac{t^2}{m^2}\xi_i^2\right)} \right.$$
$$- i\left\{\frac{ht}{2\pi m x_i^2}\left(x - x_m - \frac{1}{2}gt^2\right)^2 + \frac{4\pi}{h}\xi_m x_i^2 \right.$$
$$\left.\left(x - x_m - \frac{1}{2}\frac{\xi_m}{m}t\right) + \frac{8\pi^3}{h^3}gm^2\xi_m x_i^6\right\} \div 2\left(x_i^2 + \frac{t^2}{m^2}\xi_i^2\right) \tag{6}$$
$$\left. - i\frac{2\pi}{h}gmt(x - x_m) + i\frac{\pi}{3h}g^2mt^3 \right]$$

where $i = \sqrt{-1}$, t = time, ξ_i is given by

$$x_i\xi_i = \frac{h}{2\pi}, \tag{7}$$

and C, x_i, x_m, ξ_m and g are constants whose meaning we shall presently discover (exp y means, of course, e^y). The reader can verify if he wishes that this rather complicated expression for ψ really is a solution of (5); the method of obtaining it is an indirect one.[7, 9] The corresponding probability function for position is:

$$\psi\bar{\psi} = C\bar{C}\left(1 + \frac{h^2t^2}{4\pi^2m^2x_i^4}\right)^{-1} \exp\left\{-\frac{\left(x - \frac{t}{m}\xi_m - \frac{1}{2}gt^2 - x_m\right)^2}{x_i^2 + \frac{t^2}{m^2}\xi_i^2}\right\}; \tag{8}$$

$\psi\bar{\psi}dx$ taken for given values of x and t denotes the probability that, if the position were precisely determined at time t, a value lying between x and $x + dx$ would be obtained. Of the constants, C must have such a value that $\int Pdx = \int \psi\bar{\psi}dx = 1$. The constant x_m is simply the "mean expectation" of value for x at the time $t = 0$, for if we put $t = 0$ in (6) we find $\int_{-\infty}^{\infty} x\psi\bar{\psi}dx = x_m$; whereas x_i is the "measure of definiteness" at $t = 0$ as defined by the equation $x_i^2 = 2[(x - x_m)^2]_m = \int_{-\infty}^{\infty} 2(x - x_m)^2\psi\bar{\psi}dx$. The measure of definiteness at time t is similarly

$$x_{it} = \sqrt{x_i^2 + \frac{t^2\xi_i^2}{m^2}} = x_i\sqrt{1 + \frac{h^2t^2}{4\pi^2m^2x_i^4}}. \tag{9}$$

If in (6) and (8) we substitute

$$x' - x_m = \frac{x_i}{x_{it}}\left(x - \frac{t}{m}\xi_m - \frac{1}{2}gt^2 - x_m\right)$$

we find

$$\psi\bar{\psi} = C\bar{C}\left(1 + \frac{h^2t^2}{4\pi^2m^2x_i^4}\right)e^{-(x'-x_m)^2/x_i^2},$$

[9] E. H. Kennard, *Phys. Rev.*, 31, 876, 1928.

in which the factor containing x' is independent of t. From this result it follows easily that the probability curve at any time t can be obtained from that at time $t = 0$, when $x = x'$, merely by stretching all abscissas in the ratio x_{it}/x_i while holding the ordinate at $x' = x_m$ fixed, then displacing the whole curve a distance $t\xi_m/m + \frac{1}{2}gt^2$ toward $+x$, and finally decreasing ordinates in the proper ratio to keep the area under the curve equal to unity. We can express this in another way by saying that the packet spreads out with time, (its diameter ultimately increasing at the uniform rate $h/2\pi m x_i$ or ξ_i/m) while at the same time the whole packet moves with a velocity consisting of a uniform component $u_m = \xi_m/m$ upon which is superposed a component with uniform acceleration g. We have thus an example of a concentrated probability packet which both moves as a whole and changes size as it goes.

THE AMPLITUDE FOR MOMENTUM.

In terms of this example we can bring out more clearly the status of velocity and momentum in the new theory. In accordance with the general discussion above, it is not immediately obvious what should be meant by "the velocity" of the particle whose position in space is described by the amplitude ψ; were we to locate the particle in order to begin an observation of velocity, we should seriously disturb it through the Compton effect and should alter completely the value of ψ. Apparently we can do nothing better than the following. Let us suppose that, as in the above example, at time $t = 0$, ψ differs appreciably from 0 only over a finite range D (in the example, say $D = 10x_i$). Let us wait a long time t and then locate the particle with precision, say at a point x_1. Then we may regard $u = (x_1 - x_0)/t$, where x_0 is any point within the region occupied initially by the packet, as the resulting observed value of the velocity, or mu as the observed value of its momentum. The value of u so found is not affected by the observational disturbance of ψ, which occurs only after we are through with the particle, and the uncertainty as to the value that ought to be taken for x_0 is only of order D/t and so can be made as small as we please by sufficiently increasing t.

In the case of free motion ($g = 0$ in the example) this

definition of velocity as an observable magnitude is quite satisfactory, for it is easily shown that the result of such an experiment is independent of the times chosen as the initial and final ones so long as the interval between them is long. But in any other case than free motion, as in our example if $g \neq 0$, the result of such an observation does depend upon the times; of course this is really only what one would expect in the case of any "accelerated" motion, but in Quantum Mechanics it leads to consequences disastrous for the concept of velocity, because the relatively long time that is necessarily required to make an observation precludes us from saying with precision to just what instant the measurement refers. The most natural view, in the author's opinion, is that velocity and momentum are conceptions which really possess a simple immediate significance as physically observable magnitudes only in the specially simple case of free motion.

Even in the case of free motion, moreover, the peculiar statistical foundation of the new theory manifests itself in the circumstance that the result of such an observation of velocity or momentum can only be predicted statistically (because of the spreading out of the packet with time). Thus in the example considered above it is readily shown that with $g = 0$ we can take as the probability amplitude for the momentum, $M(\xi)$, where $\xi = mu$ and

$$M(\xi) = \frac{1}{\sqrt{h}} \int_{-\infty}^{\infty} \psi(x) e^{2\pi i h^{-1} \xi x} dx$$

$$= C \sqrt{\frac{x_i}{\xi_i}} \exp \left\{ - \frac{(\xi - \xi_m)^2}{2\xi_i^2} + \frac{2\pi i}{h} x_m \xi \right\}, \tag{10}$$

the probability that an experimenter would obtain, by the method described, a value of ξ lying within a range $d\xi$ being

$$P_\xi d\xi = M\bar{M} d\xi = C\bar{C} \frac{x_i}{\xi_i} e^{-(\xi - \xi_m)^2/\xi_i^2} d\xi. \tag{11}$$

From (11) we find that $\int_{-\infty}^{\infty} \xi P_\xi d\xi = \xi_m$, $\int_{-\infty}^{\infty} 2(\xi - \xi_m)^2 P_\xi d\xi = \xi_i^2$, so that the constants ξ_m and ξ_i appearing in (6) represent

respectively the mean expectation of value and the measure of definiteness for the momentum. The constancy of ξ_m in time is, of course, due to the absence of external forces; if $g \neq 0$, ξ_m as defined by means of (10) and (11) is found to increase at the rate of mg units per second. The mean expectation of velocity, $u_m = \xi_m/m$, is simply the velocity of advance of the whole packet, as is reasonable.

The extension of the idea of momentum as an observable quantity to cases of accelerated motion, as we have seen, presents difficulties. For theoretical purposes, however, it is often convenient to generalize the mathematical formulas and to define as the probability amplitude for any momentum p corresponding to a generalized coördinate q:

$$M(p) = \frac{1}{\sqrt{h}} \int \psi(q) e^{2\pi i h^{-1} pq} dq, \tag{12}$$

with a suitable generalization for the case of many degrees of freedom. It can then be shown[8] that, if q_i and p_i are the "measures of definiteness" of q and p respectively, in general

$$q_i p_i \geqq \frac{h}{2\pi}. \tag{13}$$

This equation represents a very general case of Heisenberg's Indetermination Principle.

MAGNITUDE OF THE QUANTUM INDEFINITENESS.

At this juncture it may be illuminating to turn aside for a moment from the general discussion and to give some illustrations of the actual magnitude of the quantum indetermination. This feature of the theory only makes itself felt, of course, in very small-scale phenomena.

For example, a bacterium weighing 10^{-12} g. might be represented by a packet like that expressed by eq. (6) with $x_i = .03\ \mu$, so that its position is fixed initially within about $0.1\ \mu$; then ξ_i equals 3.5×10^{-22}, so as to make $x_i \xi_i = h/2\pi = 1.04 \times 10^{-27}$, and the indefiniteness in velocity is under $10^{-5}\ \mu$ per second. To double in size, such a packet will require a time t that may be found by putting $x_{it} = 2x_i$ in

(9), from which $t = \sqrt{3}mx_i/\xi_i$ or more than four hours. Similarly, for the smallest water drop visible to the naked eye the doubling might easily take a million years. Thus such large bodies as these can be handled very well by the approximate methods of classical mechanics. On the other hand, if a molecule of air were fixed in position within the length of its own diameter (say $x_i = 6 \times 10^{-9}$ cm.), its velocity would necessarily be indeterminate by at least 3000 cm./sec. and its wave-packet would enlarge by at least one molecular diameter every 10^{-11} seconds; whereas the packet for an electron defined in position within the radius of the smallest Bohr orbit for the hydrogen atom (say, $x_i = 3 \times 10^{-9}$ cm.) would at least double in size in 1.5×10^{-17} sec. or in less than a tenth of the time required for one revolution around its orbit in the old quantum theory. Thus, according to the new theory, even if we could locate a particle of atomic dimensions within a distance equal to its own diameter we should lose track of it again very quickly indeed.

MONOCHROMATIC WAVES OF ψ.

Returning now to our general train of thought, let us consider the very important case in which we have to deal, not merely with a "wave-disturbance," but with actual "waves" analogous to the monochromatic wave-trains of optics. In eq. (6) let us put $g = 0$ so as to have a case of free motion, and let us make x_i very large; ψ can then be written

$$\psi = \gamma(x, t)e^{2\pi ih^{-1}(Wt - \xi x)}, \tag{14}$$

where $W = \xi^2/2m$ (and represents the energy), ξ is written for ξ_m, and γ is a function that is almost constant in space and in time but nevertheless manages to vanish at infinity and to make $\int \psi\bar{\psi}dx = 1$. The momentum is now confined practically to the value ξ_m (for which we have written ξ), for in (10) $\xi_i = h/2\pi x_i$ and is small. The packet will still move along with a speed $u_m = \xi_m/m$ or $u = \xi/m$; and its motion is in this case not much blurred by spreading, for by (9) the increase in its diameter in time t is only of the order of

$$x_{it} - x_i = \frac{h^2t^2}{4\pi^2m^2x_i^2}\frac{1}{x_{it} + x_i} < \frac{h^2t^2}{8\pi^2m^2x_i^3},$$

whereas the time required for the packet to traverse a distance nx_i is $t = nx_i/u$, so that with both n and x_i large the quantity $(x_{it} - x_i)$ may still be small and the packet can therefore move over many times its own diameter (n large) before spreading appreciably. Such a packet represents a particle whose velocity is very closely defined, while its position is largely indefinite and yet confined within sufficiently close limits so that it can be regarded, on the relatively large scale of physical experimentation, as located at a moving geometrical point.

The fact that such a packet moves approximately as a unit suggests that whenever we have a stream of cathode rays or of β-particles of approximately uniform speed we can suppose each one to be accompanied by a finite train of waves of this type (if it were necessary to take account of interaction between the particles we should have to replace all of these separate packets by a single ψ containing the coördinates of all of the particles). A complete treatment of the situation would involve a quantum-mechanical analysis of the system producing the cathode-rays, but this is too complicated to handle, so we are compelled to proceed as in optics and infer the character of the wave-disturbance from what it is found to do. The length of the train will depend upon how closely the speed of the particle is determined by the experimental conditions. When the particle strikes an obstacle, like the electrons falling upon the crystal in the experiments of Davisson and Germer, its wave-train is scattered to the four winds and produces diffraction and interference patterns along the path of its flight. Since, however, the waves are only probability waves, this scattering implies no division of the electronic charge and energy; it is only that the position of the charge and energy becomes increasingly indefinite.

The monochromatic waves that can thus be supposed to accompany electrons of definite speed are usually extremely short. The general formula for their length is, from (14), $\lambda = h/\xi = h/mu = h/\sqrt{2mW}$, W being the kinetic energy of an electron; this gives $\lambda = 1.3 \times 10^{-7}\sqrt{1/V}$ where V is the energy in equivalent volts and λ is in centimeters. Thus even millivolt electrons have a wave-length of only about 4×10^{-6} cm. or the same as very short ultra-violet light and

the electrons of a hundred volts or so with which Davisson and Germer worked have wave-lengths of X-ray magnitude. The study of electron diffraction and interference thus requires a technique similar to that employed for X-rays and we can understand very readily why these phenomena were only discovered very recently (and under the stimulus of the new theory, as a matter of fact).

Probably a theory for photons, similar to the above for electrons, will eventually be worked into satisfactory shape, the field vectors being interpreted as, or replaced by, a probability amplitude for the photons. The explanation of an interference experiment like that described at the beginning of this article, whether performed with electrons or with photons, will then be somewhat as follows. Each wave-packet representing a particle passes through the apparatus according to the wave-equation, exhibiting such phenomena as refraction and interference. The wave amplitude at any point then determines the probability of finding each particle at one place or another, or of each particle's producing at each point some physical effect such as the blackening of a photographic plate. Thus the probability amplitude is propagated according to the wave-equation, but charge and energy, when they put in an appearance, do it in a single unitary act confined to a highly restricted region. Of course it will always be possible to believe, if one will, that electrons and photons are really particles of the old sort moving along definite paths and that the new theory with its essential element of probability merely expresses a hopeless limitation in our knowledge of their exact motion. So long, however, as it is impossible either to eliminate this element of probability, or to imagine exact laws according to which their true motion might be taking place, it seems likely that physicists generally will prefer the simpler view according to which electrons and photons are entities to which the concept of continuous motion can be applied only with an essential degree of vagueness.

Before leaving the subject of electron streams it should be mentioned that theorists have usually worked with a wave-function simpler even than (14), viz.:

$$Ce^{2\pi i h^{-1}(Wt-\xi x)}, \tag{15}$$

where C is an arbitrary constant. Such a function cannot, of course, constitute the amplitude for a particle, since the condition that $\int \psi\bar{\psi}dx = 1$ would require that $C = 0$; it represents only an ideal mathematical case, analogous to the exactly harmonic wave-train of optics, which is of necessity of infinite length. The general solution of (4), the wave-equation for a free particle in one dimension, can be written subject to certain restrictions in the form

$$\psi = \int_{-\infty}^{\infty} M(\xi) e^{2\pi i h^{-1}(Wt - \xi x)} d\xi,$$

where $M(\xi)$ is the amplitude for momentum as defined above, just as the wave function in optics can be written as a Fourier integral in which the square of the coefficient in the integrand represents the intensity at a given point in the spectrum. The chief application of the simple form (15), originated by Born, is to problems in which the velocity of a particle is pretty definite while its position is (on the molecular scale) indefinite, as in calculating the ionization produced by a stream of cathode particles in a gas. A somewhat different interpretation from the meaning usually given to ψ is employed in such cases, however; the quantity

$$S_x = \frac{h}{4\pi mi}\left(\psi \frac{\partial \bar{\psi}}{\partial x} - \bar{\psi}\frac{\partial \psi}{\partial x}\right) = C\bar{C}\xi/m \qquad (15a)$$

is taken to represent the stream density of the particles or the number of them that cross unit area per second in the direction of the x-axis. This interpretation is closely analogous to the use of the conception of the intensity of a beam of radiation in optics, and the vector S is the exact analogue of Poynting's vector. The interpretation is justified by the fact that in the case of one particle the integral of the normal component of S over any closed surface can be shown to equal the rate of decrease in the integral of $\psi\bar{\psi}$ throughout the enclosed volume; if we are dealing with a large number N of particles moving independently, they can all be represented by the same wave-packet with ψ multiplied by $\sqrt{N}$, so that $\int \psi\bar{\psi}d\tau = N$, and the integral of the normal component of the vector S over any closed surface will then equal the de-

crease in the probable number of particles to be found within the surface and so may be regarded as the rate at which they are crossing it in an outward direction.

The method of handling ionization problems is therefore as follows.[10] One assumes a wave-train of the type (15), representing a stream of electrons moving in a definite direction with kinetic energy W. When these fall upon an atom the waves are scattered in all directions and this scattering is calculated by solving the wave-equation for the combined system of electron and atom. The scattered waves moving out in any chosen direction from the atom are then found to consist of one or more terms like that in (15), and the vector S for each term, divided by its value for the incident wave-train, is taken to represent the fraction of the electrons falling upon unit area in the incident stream which are scattered per unit area in the chosen direction and with energy corresponding to that term.

Experiments upon the simple elements of Quantum Mechanics, which we have been considering so far, are, it must be confessed, very scarce as yet because of their difficulty. We must conclude, therefore, with a brief consideration from the mechanical standpoint of the fields in which the new theory has scored its greatest triumphs. This is, first of all, the field of atomic energy levels, and secondarily that of radiation intensities.

ATOMIC ENERGY LEVELS AND QUANTUM STATES.

In approaching the subject of atomic energy from the mechanical side, we have first to raise the question, what is to be meant by energy as an observational quantity. In classical theory the energy was given as a function of the positions and velocities of the particles, but if we employ this definition in the new theory a precise measurement of energy becomes impossible because positions and velocity cannot be measured with precision simultaneously. Energy thus exhibits quantum-incompatibility with position and velocity (or momentum) taken together, and we are thrown back upon a consideration of direct experimental methods for its measurement. Now it must be admitted that no such

[10] M. Born, *Nachrichten der Ges. der Wiss. zu Göttingen*, 146, 1926.

method has ever actually been used, but an excellent clue is furnished by the Stern-Gerlach experiment, in which the deflection of an atom moving through a non-homogeneous magnetic field is made to measure the magnetic moment of the atom and so also its angular momentum, which is a mechanical quantity in the same general quantum status as the energy. It seems reasonable to assume that an atom might similarly be subjected to forces proportional to its energy, and its resulting displacement, or some indirect effect of this, might then be taken as measuring this magnitude; for instance, if gravitational mass and energy really are proportional to each other, we might "weigh" the atom.

The analysis of such an experiment, in which an atom is assumed to move under a force proportional to its internal energy, shows [11] that the result is a splitting up of the probability packet. We can write

$$\psi' = \psi_c(x, y, z, t)\psi(q, t) \tag{16}$$

where x, y, and z are the coördinates of the center of mass of the atom and q stands for all of its internal coördinates; for convenience we have written ψ' where ψ would ordinarily stand so as to reserve the symbol ψ for the internal factor alone. $\psi(q, t)$ is a solution of Schrödinger's equation for the atom relative to its center of mass, viz.:

$$\frac{h}{2\pi i}\frac{\partial \psi}{\partial t} = H\left(-\frac{h}{2\pi i}\frac{\partial}{\partial q}, q\right)\psi, \tag{17}$$

where H is the Hamiltonian with the part referring to the center of mass omitted. Now the general solution of this last equation can be written

$$\psi = \sum_n c_n e^{2\pi i h^{-1} W_n t} u_n(q, W_n) + \int \gamma(W) e^{2\pi i h^{-1} W t} u(q, W) dW \tag{18}$$

where $u_n(q, W_n)$ or $u(q, W)$ is one of those solutions of the Schrödinger equation for the atom,

$$H\left(-\frac{h}{2\pi i}\frac{\partial}{\partial q}, q\right)u = Wu, \tag{19}$$

[11] E. H. Kennard, *Phys. Rev.*, 31, 876, 1926.

which is single-valued and finite everywhere over the physically admissible range for the q's; only values of W for which this special type of solution exists ("Eigenwerte" or characteristic numbers) and the solutions corresponding to them (characteristic functions) are to be employed, and the sum in (18) extends over such of these values of W as form a "discrete spectrum" while the integral extends over any continuous ranges of such values of W forming a "continuous spectrum." The u's have the property that $\int u_n \bar{u}_m dq = 0$ if $n \neq m$, and the arbitrary constant which they contain as a factor is to be so chosen that $\int u_n \bar{u}_n dq = 1$ (with a suitable modification for the continuous spectrum). The coefficients c_n or $\gamma(W)$ are arbitrary; if ψ is given at a certain time t_0, these constants can be adjusted to make ψ reduce to the given function of the q's at time t_0. Now what would happen in the measurement of the energy is that the various components of the packet, as represented by the separate terms in (18) corresponding to different values of W_n, or by the parts of the integral corresponding to different elements dW, would undergo different displacements in the x, y, z space of the center of mass, with the result that the whole packet would be spread out into a spectrum in this space according to the values of W. The probability of finding the system in the place belonging to a given W_n or dW is easily shown to be $C_n \bar{C}_n$ or $\gamma\bar{\gamma} dW$, respectively. Now in a special case it might happen that the whole packet came to a single spot in the spectrum; in such a case we should naturally regard the corresponding value of W as an observed value of the energy. Otherwise we should have a probability curve for the energy; by analogy with the meaning of ψ and $M(\xi)$ we can call c_n or $\gamma(W)$ the probability amplitude for energy.

We have thus obtained both a realization of atomic energy as an observable physical magnitude and a method of calculating its possible values from the Hamiltonian function. The possible values that the energy can have are simply the characteristic values of W for the ordinary Schrödinger equation, (19), in which H denotes the Hamiltonian for the atom with its mass-motion omitted. This is the famous method of calculating energy levels which has

succeeded so wonderfully and is too well known to require further comment.

Any one term or part of the integral in (18), with the coefficient omitted so as to preserve the relation $\int \psi\bar{\psi}dq = 1$, such as

$$\psi_n = e^{2\pi i h^{-1} W_n t} u_n(q, W_n) \qquad (20)$$

or

$$\psi\Delta W = \int_{\Delta W} e^{2\pi i h^{-1} W t} u(q, W) dW,$$

thus characterizes the atom in a state in which its energy is definitely equal to a certain value W_n, or lies within the range ΔW. Such a ψ_n can be regarded as a "standing wave" of probability amplitude, albeit one of a novel sort, since the amplitude does not pulsate in magnitude but merely revolves steadily in the complex plane. The corresponding probability function for the positions of the electrons,

$$\psi\bar{\psi} = u_n(q, W_n)\bar{u}_n(q, W_n),$$

is entirely independent of time. We have thus in this case a wholly static atom. Perhaps such a result may seem paradoxical in view of the fact that under many circumstances the electrons could certainly be followed in orbital motion around an attracting center like the nucleus. The solution of the paradox lies in the fact that a single characteristic solution such as that in eq. (20) refers to an atom that has been carefully selected so as to have a perfectly definite value of the energy, and any such process of selection requires a certain lapse of time; for instance, to select an atom in the normal state, we might simply wait until the atom had almost certainly jumped spontaneously into that state. During this lapse of time any definiteness as to the electronic motions within the atom which may have existed initially gradually disappears and by the time the energy has become definitely equal to W_n all trace of definite motion is gone.

To give a simple example, for the normal state of the hydrogen atom eq. (20) takes the form:

$$\psi_1 = (\pi a^3)^{-1/2} e^{2\pi i h^{-1} W 1 t - r/a}, \qquad \psi\bar{\psi} = \frac{1}{\pi a^3} e^{-2r/a},$$

where $a = h^2/(4\pi^2 me^2)$ (a thus equals the radius of the smallest Bohr orbit), $W_1 = -2\pi^2 me^4/h^2$, and r = distance from the nucleus to the point at which ψ_1 has the value given (e in the equation stands as usual, of course, for the Napierian base!). The probability of finding the electron in a given element of volume is thus in this case greatest at the nucleus (but we must remember that the theory here given is non-relativistic and treats both nucleus and electron as a point charge, so that the true value of ψ may well be quite different just at the nucleus). Yet we should not really be likely to find the electron so close in, for the probability of finding it at a distance away between r and $r + dr$ is $4\pi r^2 \psi\bar{\psi} dr$, which has a maximum for $r = a$. The radius of the Bohr orbit thus represents in this case the most probable distance of the electron from the nucleus. The electron is, however, more likely to be found still farther away rather than nearer, it has a chance of one fifth of being found more than twice as far out, five per cent. for three times as far, and so on. In the higher states of excitation the probability distribution broadens out still more just as did the old Bohr orbits.

The picture of the atom which we thus obtain from the new Quantum Mechanics is decidedly different from the one furnished by the old quantum theory. It has something of resemblance to the static atom imagined at one time by various physicists although the indefiniteness in the location of the electrons makes the resemblance rather a superficial one. Probably the new picture will seem somewhat repellent to many thinkers, who find a more concrete model a great help in fixing their ideas; and it may be that the future will lead us again to a conception of the older sort, although it seems unlikely to be the one of Bohr, at any rate.

INTERACTION BETWEEN ATOMS AND RADIATION.

For the sake of completeness we shall now consider briefly the problem of the interaction of atoms with radiation, which embraces all the phenomena of refraction, scattering, absorption and emission. The correct method of handling such problems seems to have been discovered, but its true interpretation probably remains to be found; speaking more generally, the complete quantum theory of the electro-

magnetic field remains to be elaborated. There seems to be no reason for abandoning the hope that when this has been found the theory of radiation will turn out to be as simple in conception as is the new theory of mechanical systems, the abstruse processes of the generalized matrix theory being no longer needed. What we have at present is rather like snatches of a rocky coast seen through breaks in the fog. For simplicity we shall confine the discussion to atoms with a single electron. In one of his original papers Schrödinger takes account of the effect of radiation upon an atom, of a frequency lying outside of its absorption band, by treating the radiation simply as a force field and introducing appropriate terms into the Hamiltonian function. It then turns out that the simplest solution of the wave-equation, (17), is not a single term ψ_n but a series somewhat like (18); if we suppose for simplicity that the characteristic functions u are real, the corresponding probability function for an atom in the nth quantum state can be written, to the first order in the amplitude b of the radiation:

$$\psi\bar{\psi} = u_n^2 + 2bu_n \cos 2\pi\nu t \left\{ \sum_s \frac{\beta_{ns} u_s}{(W_n - W_s)^2 - h^2\nu^2} + \int \frac{\beta_n(W) u(q, W) dW}{(W_n - W)^2 - h^2\nu^2} \right\}.$$

the β's denoting constants characteristic of the atom and ν the frequency of the radiation. Now Schrödinger interpreted $\psi\bar{\psi}$ as actual density of electricity, normalizing his ψ so that $\int \psi\bar{\psi} dq = e$ (electronic charge) instead of unity. The harmonic term in the expression just given for $\psi\bar{\psi}$ would then mean a vibrating distribution of charge, and this vibrating electricity he assumed to radiate according to the classical laws. This assumption gives, as a matter of fact, so far as tests have yet been made, a satisfactory account of the phenomena of dispersion and scattering, so that the theory must be at least mathematically correct.

Such an interpretation of $\psi\bar{\psi}$ is obviously at variance with the standpoint taken in this paper and it does not seem to have been accepted generally by theoretical physicists except as a means of visualizing a successful rule for calculat-

ing the optical behavior of atoms. On the other hand, we have at present nothing to replace it.

A still more baffling case from the standpoint of interpretation is presented by the theory of "spontaneous emission." When an atom is definitely in a certain excited state, ψ is supposed to have a form like that in eq. (20) and $\psi\bar{\psi}$ is constant in time; according to Schrödinger the electricity should therefore be at rest and there should be no emission. Actually, however, we know that emission may occur, accompanied by a quantum jump; and it is believed, with considerable support from experimental checks, that the following method of calculation leads to correct values of the emission, or jump, probabilities for a one-electron atom. Let the amplitude for the actual state of the atom be ψ_n as given in (20), normalized as before, so that $\int \psi_n \bar{\psi}_n dq = e$, and let ψ_m be the corresponding amplitude for any other state of lower energy, W_m. Then we find that (the u's being real)

$$\psi_n\bar{\psi}_m + \bar{\psi}_n\psi_m = 2u_n u_m \cos 2\pi \frac{W_n - W_m}{h} \cdot t. \quad (26)$$

Now assume that there is a fluctuating distribution of electricity with density $\rho = (\psi_n\bar{\psi}_m + \bar{\psi}_n\psi_m)$ and calculate the classical radiation from it, divide the rate of emission of energy in this classical radiation by $h\nu_{nm}$ where $\nu_{nm} = (W_n - W_m)/h$; and the result is the probability per second that a photon of frequency ν_{nm} will be emitted and that the atom will simultaneously jump into the state m.

Such a method of calculating emission probabilities obviously has no footing at all within the frame of the Heisenberg-Schrödinger mechanics as we have sketched it. We are required to combine two amplitudes each of which in our theory represents the atom only when it has been specially selected so as to be in a definite quantum state, and it cannot possibly be selected so as to be definitely in each of two states at once. Nor does this method of treating spontaneous emission agree with Schrödinger's treatment of scattering and dispersion, for there we worked with a single ψ, consisting of a series of terms added together and normalized so that $\int \psi\bar{\psi} dq = e$, whereas here we are required to take two ψ's belonging to different states, each one normalized separately

by the same rule, and then to combine them without adding them!

The only possible conclusion to be drawn from this situation seems to be that there is a link still missing in the theory. We can see in a general way what sort of element remains to be supplied. For one thing, it will probably be necessary to work with a probability amplitude for the photon, and to deduce the Schrödinger rule for calculating radiation from a law connecting the time rate of change of the amplitude for the photon with the atomic ψ; this law must, however, be rather different from the Schrödinger wave-equation for atoms because photons can (apparently, at least) be created and destroyed. Again it may be that the series that we obtain for ψ when the atom is under the influence of radiation should be interpreted to mean that the state of the atom under these conditions is not fully defined, there being only a certain probability for each of the quantum states. Probably the most radical innovation, however, will be the one to take account of the spontaneous quantum jumps.

Several physicists have attempted to account for spontaneous changes in atomic energy (ignoring for the moment the accompanying emission of energy) by introducing into the wave-equation a term corresponding to radiation damping in classical theory, the effect of which would be to make the coefficients c and γ in (18) change continuously with time until eventually the whole expression becomes reduced to the single term referring to the normal state. Such an attempt seems, however, foredoomed to failure because there seems to be something essentially irregular about the transitions between states.

Consider, for instance, an atom in a metastable state of energy W_2, from which there is a small but finite transition-probability to the normal state of energy W_1, corresponding, let us say, to a mean life of an hour in the higher state. The transition, when it occurs, will be accompanied by the emission of a photon, and by observing this we can fix the time of the jump. Then we have this situation: before the photon comes out, the atom must surely be represented by a single term like (20), consisting of the function proper to the metastable state, while after the jump ψ must have the form that

represents the normal state. Thus we must suppose that, at a time which is itself controlled only by a probability law, ψ begins to change and quickly passes over into a different form, which it thereafter retains.

Spontaneous "jumps," therefore, are real physical occurrences and are not merely an illusion. To give an account of them it would seem as if a statistical element must be introduced into the very wave-equation itself.

ELECTRICAL AND MAGNETIC PROPERTIES OF THE ATOM.

The interpretation of $e\psi\bar{\psi}$ (with ψ normalized to unity) as density of electricity has applications even beyond the theory of radiation. The electrical field calculated from such a distribution, using the value of ψ proper to the quantum state in which the atom is assumed to be, can be shown to give a good approximation to the effect of the atom upon rather distant bodies, in so far as that effect arises in the correct quantum-mechanical theory from terms in the Hamiltonian representing the mutual potential energy associated with electrostatic forces between the constituents of the atom and of the distant body. That is to say, in order to calculate the interaction between two atoms close together one must employ the special methods of quantum mechanics, whereas if the atoms are rather far apart as compared with their diameters either one of them may be replaced approximately, as regards its electrostatic effect upon the other, by a distribution of charge $e\psi\bar{\psi}$ (with a suitable generalization for atoms containing more than one electron). The distant magnetic effects also can be obtained, it is generally believed, in a similar manner. In cases in which the density $e\psi\bar{\psi}$ varies with time, there must be a set of associated currents to move the charges about, and it is easily seen that the density of these currents is eS where S is the vector defined by the first equation in (15a) above. Now in many cases S is not zero even when $e\psi\bar{\psi}$ is constant in time; and in such cases the magnetic moment due to these currents is believed to represent the magnetic moment of the atom. In this way the diamagnetic and paramagnetic properties of bodies are treated on the new theory;[12] but it has not been possible as yet to secure very extensive comparison of results with experiment.

[12] Cf., e.g., J. H. Van Vleck, *Phys. Rev.*, 30, 31, 1927.

Such a use of $e\psi\bar{\psi}$ and S is, of course, entirely in harmony with the probability interpretation of $\psi\bar{\psi}$; the Schrödinger distributions of charges and currents are employed simply as an auxiliary idea to enable us to visualize rules of calculation which find their ultimate theoretical justification in an application of the more abstract fundamental principles of the theory. It seems rather probable that a similar view will eventually come to be taken of the role of the vibrating charge in connection with radiation phenomena.

CONCLUSION.

The new Quantum Mechanics, as we have seen, cannot yet be considered as a coherent and completed theory; its condition at the present time has been aptly compared to that of Maxwell's theory soon after its publication, only that the confusion in Quantum Mechanics is much greater because it has been the product of several minds. The parts of the theory which rest upon the firmest experimental basis are probably the part that deals with atomic and molecular energies and the treatment of electron and photon beams by means of a probability amplitude propagated according to a wave-equation. According to the theory, protons also should exhibit wave-properties in the same way as do electrons, but as yet experimental proof of this is lacking. Perhaps we must still regard it as possible that protons are fundamentally different from electrons in their mechanical properties; an experimental test of this point is urgently needed. The new Fermi-Dirac statistics finds a basis in Quantum Mechanics, but the only application of it which has met with convincing success is again one to electrons, namely, the new form given to the theory of metallic conduction by Sommerfeld.

On the whole, however, it seems certain that the Quantum Mechanics of Schrödinger and Heisenberg will form an important part of the completed theory that is to come. Whether the student of the future will also be required to learn the matrix form of the theory seems more open to question, although this form is still in favor in many quarters. Perhaps the decision on this point will come when the proper treatment of electromagnetics shall have been discovered.

SOURCES OF VISIBLE AND INFRA-RED RADIATIONS FOR DEEP THERAPY.

BY

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PROGRESS in the scientific development of radiation therapy has been seriously retarded by insufficient consideration and understanding of physical science on the part of investigators and practitioners. Radiant energy or radiations of all kinds—infra-red, visible, ultra-violet, Roentgen, radium, and others—are tools which can be adequately described only by physical units such as wave-length and intensity. The chief factor in this description is wave-length or its reciprocal —frequency. In the scientific literature of researches and practices, the spectral aspect has not been adequately treated and ovbiously in many cases has not been understood. For example, ultra-violet radiation to many persons is merely a specific thing when, as a matter of fact, it is the name applied to a great range of the spectrum including many wave-lengths, or frequencies. Its properties vary enormously with wave-length and, therefore, researches which do not definitely specify wave-lengths are of little value. This is equally true of radiations of other spectral ranges. In previous publications the writer has presented many physical data pertaining to ultra-violet [1] and visible [2] radiation. These are only a part of an adequate literature to which the medical practitioners and investigators in physiology may turn for an understanding of the physical aspects of radiation. In the present paper some of the fundamental physics of infra-red are presented with the hope that the present confusion may be partially clarified at least.

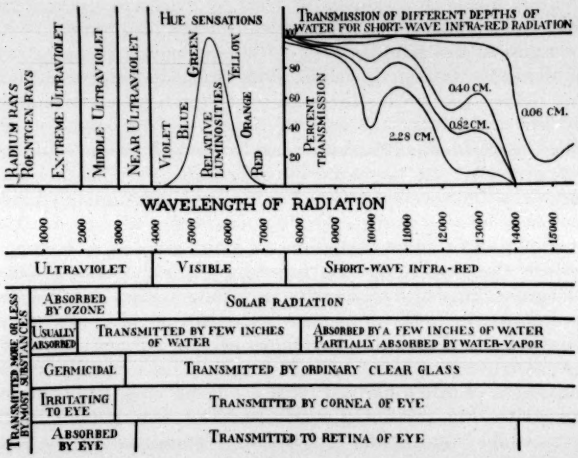
For those who are not familiar with the spectrum of radiation, Fig. 1 is inserted. The spectral range includes all wave-

[1] M. Luckiesh, "Ultra-violet Radiation," D. Van Nostrand Company, 1922,

[2] M. Luckiesh, "Color and its Applications," D. Van Nostrand Company. 1915 and 1921.

lengths from zero, through the ultra-violet and visible regions and the near infra-red. Certain properties of radiation and characteristics of the eye and of various media are also presented. In this paper wave-lengths are expressed in Ängstrom units. For example, λ7600 is the wave-length approximating the boundary line between the visible and near infra-red. Inasmuch as water plays a major rôle in the present discussion, spectral transmission curves of four different thicknesses are presented in Fig. 1 and also in other illustrations. Coblentz [3]

FIG. 1.



A diagrammatic view of various data pertaining to radiant energy. The wave-lengths are in Ängstrom units. The spectral range presented consists of the entire ultra-violet and visible radiations and the short-wave or "near" infra-red. This diagram is taken from "Light and Work," by M. Luckiesh, D. Van Nostrand Company, 1924.

has made an extensive study of the spectral transmission of water and of other media, which may interest the reader who wishes to go deeply into the subject.

In this paper the term "deep therapy" applies only to the use of visible and infra-red radiations for heating at a depth.

[3] W. W. Coblentz, "Radiometric Investigation of Water of Crystallization, Light Filters, and Standard Absorption Bands," Bul. Bureau Standards, 7, 1911, p. 619.

Deep therapy is now practiced with a large assortment of devices emitting radiations of vastly different wave-lengths covering a wide range of the spectrum. All of them cannot be most suitable for heating at a depth in the human body. Among these are many incandescent radiators ranging from relatively low-temperature carbon filaments to high-temperature special tungsten filaments. These are enclosed in bulbs of various colors or spectral-transmission characteristics. Among these are red, yellow, blue, violet and purple bulbs. Besides these there are various non-luminous radiators in use —devices heated electrically to temperatures lower than those of incandescence. It is obvious to anyone familiar with the physical aspects that most of these devices are made, recommended, and used without much knowledge of the wave-lengths of radiation involved and in some cases without even a vague conception of the physics of the subject.

Here we often find a good example of the correctness of the old adage to the effect that a little knowledge is dangerous. Vaguely it is known by many that radiation consists of "vibrations." Therefore, mysterious and even metaphysical ideas are entertained in regard to the effect of these vibrations upon human ailments. Even these vague ideas are not monopolized by the questionable practitioners but, owing to lack of knowledge, are entertained by some medical men of unquestionable character.

The one known fact of therapy is that of heating at a depth. No other fundamental cause of beneficial results is known. No direct influence of wave-length upon curative value has been found as in the case of ultra-violet radiation. Therefore, at least the physics of the subject must deal with that of the absorption of radiation by the body. Knowledge of this aspect clarifies much of the confusion now existing in regard to infra-red therapy.

Heating can be done by radiation, convection, and conduction. When a hot towel is placed upon the skin, heating at a depth is accomplished almost solely by conduction. The skin in contact with the towel may be the hottest part of the body. Some heat from the towel is lost by radiation and conduction outward. If air comes into contact with the towel, it is heated and convection currents of air carry this heat

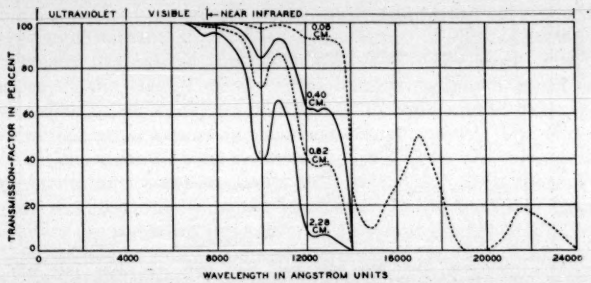
away. The liquids of the body carry some heat away by their movement, but this is not true convection and at best is perhaps a small part of the effect of the hot application.

If an electric filament lamp is held close to the body, some radiant energey enters the body. On being absorbed it is converted into heat and heating at a depth is achieved without direct contact with the hot bulb as in the case of a hot towel. Not only is this more comfortable for the patient, but greater heating at a depth can be achieved by the absorption of the radiant energy than by direct conduction. In a consideration of the physics of heating by absorption of radiant energy or radiation, it is only necessary to know the spectral transmission of the body. This is not known in actual detail but enough is known to bring order out of the present chaos: In fact, it is practicable to consider the spectral transmission of the human body to be similar to that of water, of which it chiefly consists. The present state of the art of infra-red therapy and the uncertainty of measuring the results of it do not require any greater accuracy of assumption than the foregoing. However, we may be guided further by the fact that blood transmits infra-red somewhat as red-tinted water does. The conclusions drawn in this paper would not be altered in general, and only relatively slightly in detail, if exact spectral transmission-factors of the elements of the human body were available. These conclusions will show how inconsistent the claims for the various radiators are and that many must be ruled out of consideration if the greatest heating at a depth is desired and if the greatest efficiency is a consideration.

That the human body is somewhat transparent to visible radiation is readily demonstrated by holding the hand over a light-source such as a flashlight. In the dark the hand is seen to transmit considerable light or visible radiation. It is seen that the color of the light is only a tint of red. This indicates that much of the radiation throughout the spectrum—including violet, blue, green, and yellow rays—passes through the hand. Therefore, radiation of all wave-lengths in the visible region may be considered to be more or less penetrating and useful in deep therapy. If we replaced the eyes with an instrument sensitive to the near infra-red, some of these invisible radiations would be found to pass through the hand.

Years ago in connection with a study of infra-red and the eye,[4] certain computations were made which are applicable to the present problem. In addition to these, certain modern sources have been considered, so that a view may be obtained of present radiators of a great range in temperature. Water transmits radiant energy of all wave-lengths throughout the visible and only partially in the near infra-red region. Pure water is transparent to ultra-violet energy throughout most of its spectral extent but inasmuch as sources used for deep therapy do not emit much ultra-violet, this region is not considered herewith for heating at a depth. Besides, the salts and colored compounds in the body absorb most or all of it.

FIG. 2.



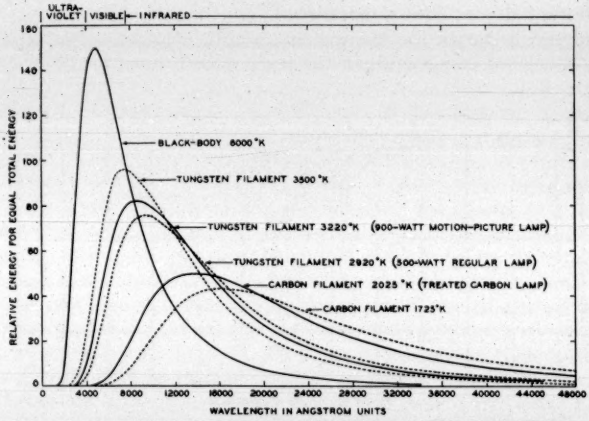
The transmission-factors of radiant energy of various wave-lengths—throughout the visible and near infra-red regions—are presented for water of four different thicknesses.

In Fig. 2 the transmission factors of four different thicknesses of water are given for radiant energy of various wave-lengths throughout the visible and near infra-red regions. It will be noted that a thickness of 2.28 cm. of water (nearly an inch) is (1) transparent to radiations of wave-lengths corresponding to the visible spectral region, (2) only partially transparent to radiations in the near infra-red, and (3) is totally opaque to radiations of longer wave-lengths than 14,000 Ängstrom units. The visible spectrum lies between approximately λ4000 (wave-length 4000 Ängstrom units) and λ7600.

[4] M. Luckiesh, "Infra-red Radiant Energy and the Eye," *Amer. Jour. Physiol.*, 50, December, 1919.

The ultra-violet extends from λ4000 toward shorter wave-lengths. The infra-red extends from λ7600 toward longer wave-lengths. Although a layer of water 2.28 cm. in thickness is completely opaque to infra-red energy greater than λ14,000, a layer 0.06 cm. in thickness transmits partially out to λ25,000 but is opaque to wave-lengths longer than this until λ80,000 is reached. This latter region is discussed later.

FIG. 3.



The spectral distributions of energy from seven incandescent solids of different absolute temperatures (Kelvin scale). The total radiant energy emitted is the same for all these radiators. As the temperature increases the amount of energy emitted in the visible and near infra-red regions increases.

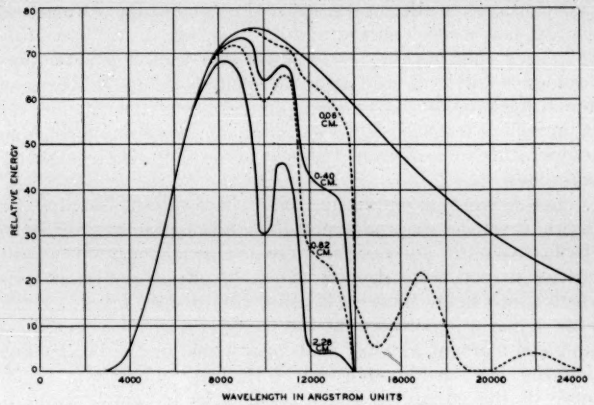
Such data as the foregoing prepare us to consider the relative value of radiators of various temperatures. In Fig. 3 are shown the spectral distributions of energy as emitted by seven incandescent solids. The black-body is a theoretical source emitting radiant energy in accordance with fundamental physical laws. Incandescent carbon and tungsten do this approximately. It is seen that as the temperature increases a relatively greater percentage of radiation is emitted in the visible and near infra-red. The areas contained within the curves and the common base-line represent relative amounts of total energy emitted by the sources at the temperatures indi-

cated. The ratio of the area between any two wave-length vertical lines, to the total area, is the percentage of the total energy in any case emitted within this range of wave-lengths. It is seen that, as the temperature increases, a greater percentage of energy is emitted by an incandescent solid between λ4000 and λ10,000 (the region of almost complete transparency of an inch of water). Thus we are ready to eliminate the low-temperature incandescent solids in favor of the high-temperature ones.

Let us pause to emphasize this. In obtaining heating at a depth, it is desirable to have as little absorption as possible in the first or outer layer of the body. This leaves more radiant energy to penetrate deeper, where by absorption it is converted into heat, thereby heating this deeper layer. From Figs. 2 and 3 it is seen that the maximum of the curve for a tungsten filament at 3500° K. is near λ7600 or the border line between the visible and near infra-red. Therefore, much more of the energy of the wave-lengths to which water is transparent, or fairly so, is emitted by this source than by incandescent solids of much lower temperature. Therefore, of all the actual sources considered in Fig. 3, the tungsten-filament lamps appear to be best suited for deep therapy. Any modification by means of colored bulbs decreases their efficacy by reducing the amount of radiant energy available. Why use a blue bulb to eliminate much of the yellow and red radiation which should be used in heating at a depth just as the blue radiations are? Still many blue bulbs are in use. Why use a red bulb to eliminate blue and green radiations when these are likewise valuable in penetrating the body? Still many of these are advocated.

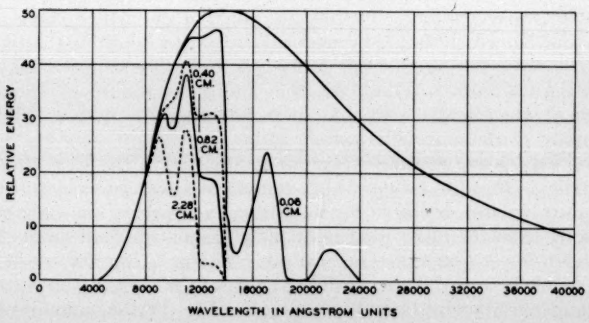
Figs. 4 and 5 make this point clearer and at the same time indicate that an ordinary high-temperature tungsten-filament lamp is more desirable for heating at a depth in the human body than the old 4 watt-per-candle carbon-filament lamp of much lower filament-temperature. In Fig. 4 the major portion of the spectral energy distribution of the present 500 watt tungsten-filament lamp is presented. The relative amount of energy of each wave-length transmitted by 2.28 cm. of water is shown in the lowest curve. The ratio of the area under this lowest irregular curve to the total area under the outer regular

FIG. 4.



The spectral distribution of energy emitted by an ordinary 500-watt tungsten lamp is shown in the upper curve. The amounts of this energy of various wave-lengths which pass through four different layers of water respectively are shown by the four irregular curves. Inasmuch as the areas represent amounts of energy, the ratio of the area under the irregular curve (marked 2.28 cm.) to the total area under the uppermost regular curve indicates the percentage of the total energy which remains unabsorbed after a depth of 2.28 cm. of water is reached.

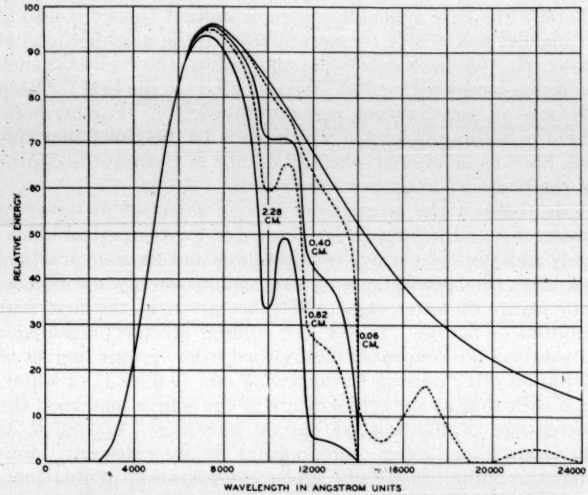
FIG. 5.



The uppermost regular curve is that of spectral distribution of energy emitted by the old 4 watt-per-candle carbon-filament lamp. The irregular curves indicate the relative amounts of energy transmitted respectively by the four layers of water.

curve is the percentage of total energy transmitted by the 2.28 cm. of water. The relative amounts of energy of various wave-lengths transmitted by the other three thicknesses of water are also shown and the same process applies to them.

FIG. 6.



Showing the spectral distribution of energy emitted by solid tungsten at 3500° K. (near its melting point) and the relative amounts of energy of various wave-lengths transmitted by the four layers of water respectively.

In Fig. 5 the same physical basis is given for the old carbon lamp—still unjustifiedly used in deep therapy. The ratio of the area under the irregular curve to the total area under the outside regular curve (spectral energy distribution for a 4 watt-per-candle carbon filament) is seen to be much smaller than in the case of the 500-watt tungsten-filament lamp. This means that in the case of this tungsten-filament lamp more energy remains unabsorbed after 2.28 cm. of water have been penetrated than in the case of the carbon lamp. Therefore, the former is the better radiator for heating at a depth.

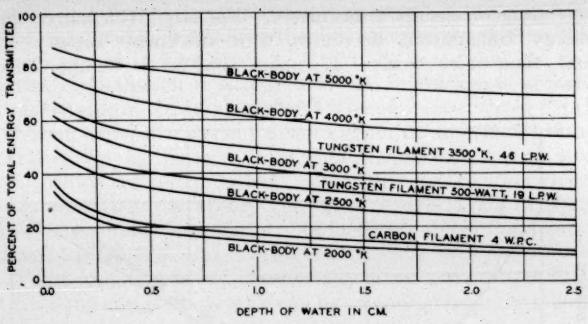
The foregoing point is further emphasized by Fig. 6, in

which the source of radiation is tungsten at 3500° K.—near the melting point. Here it is seen that the area under the lowest irregular curve (which shows the spectral transmission of 2.28 cm. of water for the radiation from this source) is a still larger percentage of the total energy emitted by the source than in the case of Fig. 4. This indicates that for deep therapy the best available source of radiant energy is a solid of highest practicable temperature up to the neighborhood of 5000° K. Of the electric filament lamps, the high-efficiency tungsten lamps with colorless glass bulbs are the best for deep therapy in the light of present knowledge. Furthermore, there is nothing unknown which leads to the belief that this will have to be altered when all details of heating the depths of the body are known.

In Table I the amounts of energy absorbed in layers of water of four thicknesses are presented for theoretical black-body radiators of various temperatures and for four practical sources. The percentages of total radiant energy absorbed in four layers of water respectively are given in the first four columns of figures. In the last column are the percentages of total energy remaining unabsorbed from various sources of radiation after passing through 2.28 cm. (0.9 inch) of water. It is seen that as the temperature of the source increases, the percentage of unabsorbed energy increases. Inasmuch as the source of lowest temperature is incandescent, non-luminous radiators are not under consideration in this comparison. Furthermore, it is seen that, of the practicable incandescent sources available, the regular high-efficiency gas-filled tungsten lamps are best. If any bulb besides a colorless one were used, the value of the lamp for deep therapy would be reduced whether the bulb were blue, violet, purple, red or any other color.

Inasmuch as the body consists largely of water, these values and conclusions are indicative of the actual and tenable ones. Any salts in solution cannot appreciably alter the general conclusions or the order of rank of these sources. Finally, the general reddish tint of the bodily tissue cannot possibly reverse the order of rank.

FIG. 7.



Showing the relation between the transmission-factor, or total energy transmitted, and the thickness of water for the nine sources considered in Table I.

TABLE I.

Percentage of Radiant Energy, from Various Sources, Absorbed in Various Layers of Water.

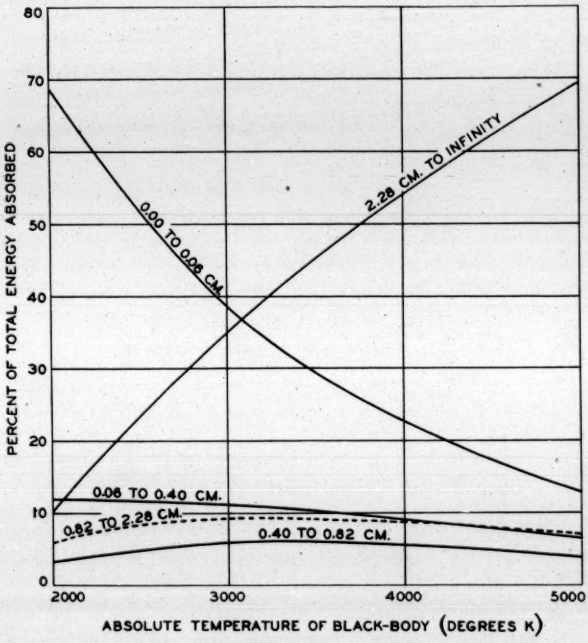
Source.	Percentage of Total Energy Absorbed in Water of Depth.				Remaining Unabsorbed at Depth of 2.28 cm.
	0.06 cm.	0.40 cm.	0.82 cm.	2.28 cm.	
Black-body 2000° K.	68.8	80.6	83.8	89.7	10.3
" " 2500° K.	51.7	63.3	68.3	76.7	23.3
" " 3000° K.	38.5	49.8	55.7	65.1	34.9
" " 4000° K.	22.8	31.7	37.2	45.9	54.1
" " 5000° K.	13.0	19.6	23.4	30.4	69.6
(*a*) Carbon filament, 2090° K.	64.1	77.3	81.0	87.9	12.1
(*b*) Tungsten filament, 2500° K.	50.4	64.5	70.5	80.0	20.0
(*c*) " " , 2920° K.	46.5	58.0	62.2	70.5	29.5
(*d*) " " , 3500° K.	33.6	43.6	48.4	56.4	43.6

(*a*) Old treated carbon-filament; 4 watts per candle.
(*b*) Approximating regular vacuum tungsten lamps; 9 lumens per watt.
(*c*) Actually a 500 watt-gas-filled tungsten lamp; 19 lumens per watt.
(*d*) Near the melting point of tungsten; 46 lumens per watt.

Fig. 7 presents a graphic view of the data found in Table I. This further emphasizes the superiority of high-temperature incandescent solids over low-temperature ones for penetrating water, or on the reasonable assumption upon which this discussion is based, for deep therapy. Here the horizontal scale

is thickness of water in centimeters and the vertical scale is in percentage of energy transmitted. For any given source the energy transmitted decreases with thickness—rapidly at first, then more slowly. It is seen that as the temperature

FIG. 8.



The percentage of total energy absorbed by water of various depths depends upon the temperature of the source of the radiant energy.

increases the curve is higher in the diagram, indicating that water is more transparent to radiations from high-temperature sources than from low-temperature ones.

In Fig. 8 are presented the percentages of total energy absorbed by layers of water at various depths for incandescent

solids of different temperatures from 2000° K. to 5000° K. The practicable incandescent-solid sources are within this range as seen in Figs 2 and 7. It will be noted that the first layer, even though it is only 0.06 cm. in thickness, absorbs considerable of the total energy. The next layer (from depth 0.06 cm. to depth 0.40 cm.) absorbs about twice as much of the total energy as the next layer (from depth 0.40 cm. to 0.82 cm.) which is slightly thicker. By consulting the other layers it is seen that the absorption of successive layers of equal thickness diminishes rapidly. In fact, this absorption follows a well-known law.

The intensity of radiation of any given wave-length after traversing any given depth d can be computed from the following equation:

$$I = I_0 e^{-xd},$$

where I_0 and I are the original and the final intensities respectively, e is the base of Naperian logarithms, and x is the extinction-coefficient. This can be further simplified for the purposes of computation into,

$$\log T = -xd \log e,$$

where T is the transmission-factor, e equals 2.7183, and $\log e$ equals 0.4343. If I_0 is taken as unity, I is numerically equal to the transmission-factor for the thickness d of the substance, for radiant energy of a given wave-length. In this manner many computations can be made throughout the spectrum and for many thicknesses of media. Years ago in connection with spectral work the writer devised a short-cut graphic method of making one set of computations or measurements answer for an infinite range of thicknesses.[2] This method is now widely used and may be of interest to those who would pursue this subject further.

In Fig. 7 it is interesting to note that as the temperature of the source increases to 5000° K., the percentage of total energy remaining to be absorbed in the layer from depth 2.28 cm. to depths beyond increases. This emphasizes again that among the incandescent solids a high-temperature source is desirable. Inasmuch as temperatures of incandescent solids of 5000° K. or more are not practicable at present, the data are not presented for higher temperatures.

Non-luminous bodies, heated electrically or otherwise to temperatures much less than that of incandescent solids, may also be judged from the basis of physics. It is known that water is transparent to radiant energy between λ80,000 and λ500,000—a range in wave-length represented in microns as from 8μ to 50μ. In Fig. 2 it is seen that the wave-length of maximum radiation shifts toward shorter wave-lengths as the temperature of an incandescent solid is increased. Conversely the shift is toward longer wave-lengths if the temperature is decreased.

The position of the maximum of the curve of spectral distribution of energy may be readily computed for the theoretical black-body from the well-known law:

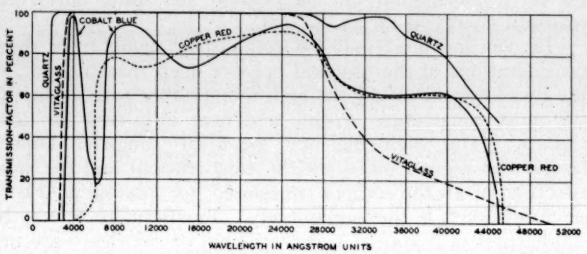
$$\lambda_m T = C,$$

where λ_m is the wave-length of maximum radiation, C is a constant, and T is the absolute temperature. When the wave-length is expressed in Ängstrom units and the temperature is expressed on the Kelvin scale, the value of the constant C is 28,900,000. This is known as Wein's displacement law and the accuracy of the law may readily be tested by the reader by multiplying λ_m and T in each case in Fig. 2. When the wave-length is expressed in any other unit the constant C is varied accordingly. For this purpose it will be recalled that one micron (μ) equals 1000 millimicrons (mμ) or 10,000 Ängstrom units. The Ängstrom unit is convenient in the ultra-violet and visible, but becomes unwieldly in the infra-red where the micron is more satisfactory. However, to avoid confusion the Ängstrom unit has been used throughout this paper.

Obviously, if non-luminous radiators are to be effective, they must radiate appreciable energy between λ80,000 and λ500,000 (8μ and 50μ). If they are to be efficient, much of the total energy emitted must be of wave-lengths within this range. Simple computations of the wave-length of maximum radiation by means of Wein's displacement law reveal the temperatures which non-luminous radiators must have if they are efficiently to supply radiation which will penetrate the body as in the case of that emitted by incandescent solids of high temperature. The wave-length of maximum radiation must

be between λ80,000 and λ500,000. These simple computations reveal the temperature range for such sources to be from 360° K. to 60° K. The upper limit is 87° C. or about the temperature of a hot-water bag. The lower limit of temperature is −213° C. or 213 degrees below zero on the Centigrade scale. Hot-water bags have often been effectively used, but a source having a temperature 213 Centigrade degrees below the freezing-point of water has not been seriously pro-

FIG. 9.



The transmission-factors of representative glasses for radiant energy of various wave-lengths.

posed by anyone, notwithstanding the fact that such a recommendation has as much scientific foundation as many of the therapeutic devices sold at a considerable price and in appreciable numbers. Therefore, it is seen that possibilities among non-luminous sources are confined to temperatures in the neighborhood of hot-water bottles. This brings us back to the hot towels, water bottles, etc., with all their discomfort, inconvenience, and lack of accurate control which modern deep therapy has largely eliminated.

It is seen that the foregoing considerations exclude non-luminous hot radiators from the class of incandescent solids as efficient and effective sources for deep therapy. Still, there are many costly devices of this kind in use. Whatever beneficial results may be obtained by them must result from heating the surface of the body just as a hot towel does. Actually heating at a depth then is achieved by the conduction of heat into deeper layers of the body. There can be little or no direct penetration of the radiant energy as in the case of

radiation from high-temperature incandescent solids such as the tungsten-filament lamp.

Owing to the questions which so often arise in regard to the spectral transmission of glasses in the infra-red regions, a few examples are presented in Fig. 9. It is seen that they are fairly transparent to the near infra-red, but that their transparency decreases in general beyond $\lambda 30{,}000$ (or 3μ). Even colored glasses have fairly high transmission-factors in the near infra-red. The clear glasses, including quartz, are almost perfectly transparent out to $\lambda 28{,}000$ and quite effectively transmit energy as far as $\lambda 40{,}000$.

In conclusion, it may be emphasized again from these considerations of the physical basis of deep therapy that, of the incandescent solids, the high-temperature sources supply the most penetrating (visible and near infra-red) for deep therapy. The high-efficiency gas-filled tungsten-filament lamps with regular bulbs are the most efficient and the most effective, of all the sources considered, for heating at a considerable depth in the human body. Furthermore, it may be emphasized that special bulbs—colored or colorless—are unnecessary and even unjustifiable and that colored bulbs reduce the efficiency and efficacy of incandescent solids for deep therapy because they absorb some of the penetrating radiation. These conclusions are drawn from a consideration of heating at a depth as being the only direct cause of beneficial results of such therapeutic practice as yet known.

VOLTAGE-INTENSITY RELATIONS OF 29 LINES OF THE MERCURY SPECTRUM.

BY

PAUL B. TAYLOR, A.M., Ph.D.,

Member of the Institute.

HERETOFORE the principal data of the spectroscopic theorist have been wave-lengths. With the growth of the quantum theory, increasing use is being made of line intensities. The measurements here reported on the variation with voltage of the intensities of most of the bright lines in the photographic spectrum of the mercury arc show a number of interesting relations of the sort that modern theory proposes to treat. No exact explanation of these relations is here attempted, as the conditions of operation of the commercial arc which was used were not sufficiently simple to warrant it.

The variation of intensity of lines of the mercury spectrum with the voltage across the arc has been studied by Athanasiu [1] and by Harrison and Forbes [2] both using radiometric methods. Valasek [3] studied the mercury spectrum using the photographic method. This last method has been used in the present investigation. It has the advantage that greater resolution of lines may be secured and faint lines (unless masked) may be measured as easily as strong lines. The lines on which measurements are here recorded are the 29 most prominent lines in the arc spectrum between 2378 and 4108, with the exception of the region about 2537 where the absorption of 2537 and the attendant bands complicate measurement. All but three of the lines belong to known series.

The source of radiation was a "Uviarc." This is a mercury arc in a straight vertical quartz tube 1 cm. in diameter, 16.0 cm. between electrodes. It was operated on rectified A.C. from a mercury arc rectifier. The current was maintained at 4.0 amperes by cooling the arc uniformly over its

[1] Athanasiu, *Compt. Rend.*, 178, 2071, 1924.

[2] Harrison and Forbes, *J. Opt. Soc. of Amer. and R. S. I.*, 10, 1, 1925.

[3] Valasek, *Phys. Rev.*, 29, 817, 1297.

entire length with a blast of air. The voltage across the arc was varied from 23 to 118 volts, 23 volts being the lowest voltage at which the arc could be maintained steadily. A thermocouple bound to the tube and shielded from the air blast gave a rough measure of the temperature of the arc. The arc was always brought to a steady state (current and temperature steady) and allowed to remain so for several minutes before making an exposure.

Light from the center of the arc was focussed by a quartz collecting lens on the slit of a Hilgar type E quartz spectrograph of 60 cm. focal length, a camera shutter being placed at the collecting lens. The shutter was electrically operated by a pendulum, the exposures being uniformly of one second duration. The intensity of exposure was regulated by a series of stops placed in the spectrograph close to the prism. The apertures in these stops varied from 6.73 sq.cm. to .0647 sq.cm. The width of the apertures was kept at 2.59 cm. in order to preserve the resolving power of the spectrograph (the slit was vertical).

In order to make the densitometer readings as reliable as possible, the spectrometer slit was kept fairly wide, .3 mm. This produced a line on the plate varying from about 10 A width at 4000 A to 1 A at 2300 A. With this width triplets were not always completely resolved. However, the results show that all the lines of a triplet maintain fairly closely the same relative intensity and, since the strongest line is usually considerably more intense than the others, it is believed that the lack of resolution has not introduced errors. Measurements on multiplets too close to be resolved are attributed to the strongest line.

All the spectra used in this series of measurements were obtained on three Eastman No. 36 plates backed with black paper. They were developed together in 1 : 100 Rodinol 17 minutes at 21° C.

The photographic densities of the lines were measured on a densitometer designed by Barnes and Fulweiler.[4] In it the plate moves slowly past a slit somewhat narrower than the lines to be measured. The density of a line was taken as the logarithm of the ratio of the galvanometer deflection

[4] Barnes and Fulweiler, *J. O. S. A. and R. S. O.*, 15, 331, 1927.

produced by an unexposed portion of the plate to that produced by the line. In some cases the unexposed portion was taken immediately above or below the line, but in most cases it was permissible to use a portion at the side of the line, for, in general, the continuous spectrum and scattered light were too faint to produce appreciable blackening. The deflection for clear plate was about 30 cm.; the densest lines gave deflections of only a few millimeters. Readings of small deflections were made to a tenth of a millimeter.

Intensity-density curves were first prepared from a series of spectra taken at a constant voltage, 23 volts, but with different stops, the logarithm of the stop area being taken as the intensity. A separate intensity-density curve was drawn for each line. It was found that the curves of faint lines were not exactly parallel to those of neighboring strong lines. This was perhaps due to the tendency of strong lines to exhaust the developer about them in development. Below a density of .2 the curves are very flat (1 to 3 slope) and so cannot be used with great precision to find intensities. Above a density of 1.5 precision again falls off as the densitometer deflections are so small.

For a line in a spectrum excited at other than 23 volts, the desity of the negative is measured and the intensity corresponding found on the I-D curve of that line. This intensity less the logarithm of the area of the stop used to photograph the spectrum gives the logarithm of the ratio of the brightness of the line at the exciting voltage to its brightness at 23 volts, and will be referred to simply as the intensity (or logarithmic intensity) of the line at that voltage. On this basis the logarithmic intensity of all lines is zero at 23 volts.

The over-all precision of the measurements can best be inferred by observing how closely the observed points lie on a smooth curve in the accompanying voltage-intensity graphs. Under the most favorable conditions of photographic density much greater precision was obtained than is here indicated in general, but these conditions could not always be met, for the range of densities in a spectrum is very great. Two density determinations of the same plate and the same spectrum showed an average difference of .007 for densities

TABLE I.

First Subordinate Series (Triplet).

Volts per cm.	.75	1.16	1.25	1.31	1.72	1.81	2.76	3.41	4.87	5.09	5.16	6.69
Temperature (° C.)	63	150	165	165	185	224	240	350	390	400	400	450
Vapor Pressure (atm.)	.00004	.004	.007	.007	.014	.07	.07	.90	1.60	2.01	2.01	4.25
					Log Intensity.							
Designation.	Wave-Length.	Photographic Density at .75 volts/cm.											
p_1d_1—$3d_1$	3650	2.15	0	.23	.28		.40	.59	.73	1.09		1.61	1.49	
p_1d_1—$4d_1$	3022	1.298	0	.54	.52		.86	1.17	1.26	1.51		2.07	1.93	
p_1d_1—$5d_1$	2803	.675	0	.74	.75	.82	1.04	1.06	1.50	1.85	2.15	2.17	2.16	2.38
p_1d_2—$3d_2$	3663	2.06	0	.05	.11		.32	.40	.73	1.00		1.45	1.38	
p_1d_2—$4d_2$	3026	.565	0	−.02			.27	.38	.78	.99				
p_2d_2—$3d_2$	3126	1.96	0	.16	.25		.45	.51	.87	1.10		1.46	1.46	
p_2d_2—$4d_2$	2652	1.233	0	.26	.27	.38	.55	.60	1.05	1.28	1.65	1.72	1.73	1.96
p_2d_2—$5d_2$	2482	.614	0	.52	.54	.92	.81	1.35	1.61			2.03		
p_2d_3—$3d_3$	3132	2.11	0	.10	.20		.36	.36	.76			1.40	1.36	
p_2d_3—$4d_3$	2654	1.086	0	.10	.20	.21	.40	.48	.92	1.15	1.60	1.61	1.62	1.60
p_2d_3—$3d_3$	2967	1.76	0	?.25	.25	.32	.46	.51	.87	1.01		1.40	1.43	1.56
p_2d_3—$5d_3$	2378	.310	0	.51	.58		.80	.86	1.31	1.53		1.99	2.00	

TABLE II.

Second Subordinate Series (Triplet).

Volts per cm.			.75	1.16	1.25	1.31	1.72	1.81	2.76	3.41	4.87	5.09	5.16	6.69
			Log Intensity.											
Designation.	Wave-Length.	Photographic Density at .75 volts/cm.												
p_1S—$3S$	3341	1.52	0	.03	.09		.28	.37	.77	.98		1.43	1.43	
p_1S—$4S$	2925	.556	0	.08	.17	.24	.38	.55	.90	1.14	1.69	1.64	1.67	1.93
p_1S—$5S$	2760	.145	0	.40		.42	.69		1.20	1.39	1.85	1.95	1.92	2.12
p_1S—$6S$	2675	.055	0							1.36	1.81	?1.97	1.88	1.99
p_2S—$3S$	2894	1.391	0	−.06	.02	.06	.20	.27	.65	.86	1.28	1.29	1.29	1.49
p_2S—$4S$	2576	.486	0	.09	.16	.20	.39	.48	.91	1.14		1.64	1.64	
p_2S—$5S$	2447	.107	0	.43			.80		1.28	1.50		1.99	2.00	
p_3S—$2S$	4047	2.41	0		−.11			.10	?.07			.75	?.87	
p_3S—$3S$	2753	1.007	0	−.11	.02	−.02	.15		.62	.83	1.31	1.29	1.28	1.53
p_3S—$4S$	2464	.292	0	.03			.33		.86	1.08		1.56	1.60	

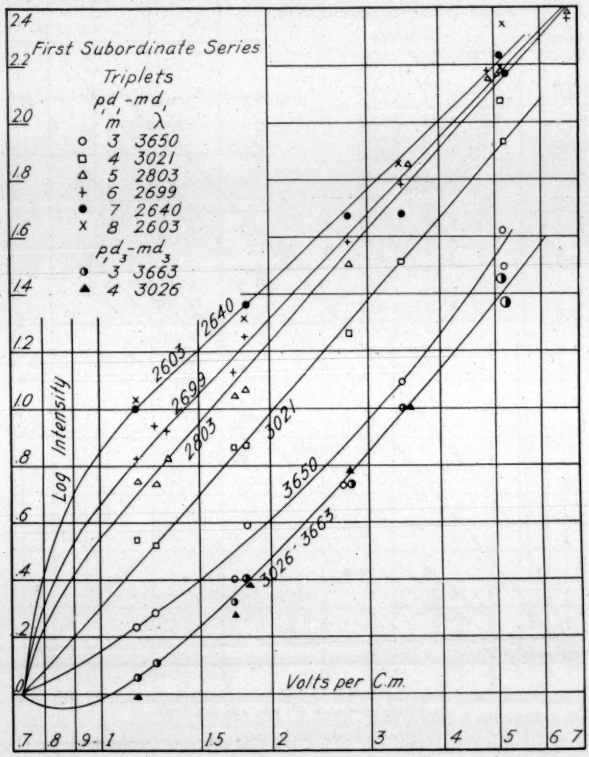
TABLE III.

Miscellaneous Lines.

Volts per cm.			.75	1.16	1.25	1.31	1.72	1.81	2.75	3.41	4.87	5.09	5.16	6.69
							Log Intensity.							
Designation.	Wave-Length.	Photographic Density at .75 volts/cm.												
							First Subordinate Series (Singlet).							
$2P$—$5D$.....	3906	.479	0	.30			.81	.70		1.32		1.82	1.87	
$2P$—$6D$.....	3704	.150	0	.43			.74			1.46		1.78	1.90	
							Second Subordinate Series (Singlet).							
$2P$—$4S$.....	4108	.162	0						.67	.92		1.48	1.46	
							Combinations between Triplet and Singlet Levels.							
$2p_2$—$2S$.....	4078	1.94	0	−.29	−.18		−.10	−.03	.21	.42		.86	.82	
$2p_2$—$3S$.....	2857	.271	0				.13		.49	.69		1.22	1.23	1.51
							Unclassified Lines.							
	2478	.02	0						1.20	1.42			2.20	
	3790	.09	0							1.45			1.84	
	3984	.142	0	.42								?1.63		

below 1 and an average difference of .05 for densities between 1 and 2. Two different spectra on the same plate of the arc

FIG. 1.

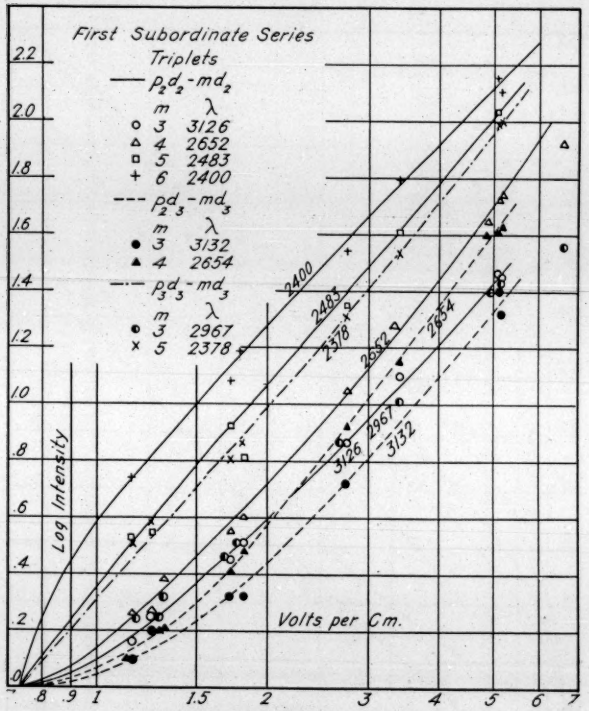


Variations of intensity of lines of the first subordinate series (triplet) of mercury with exciting potential gradient in arc, p_1d_1, p_1d_3 levels.

at the same voltage taken with different stops gave an average difference of intensity of .02 (5 per cent.). Two different spectra on different plates taken of the arc at the same voltage

and with the same stop gave an average difference of intensity of .03.

FIG. 2.

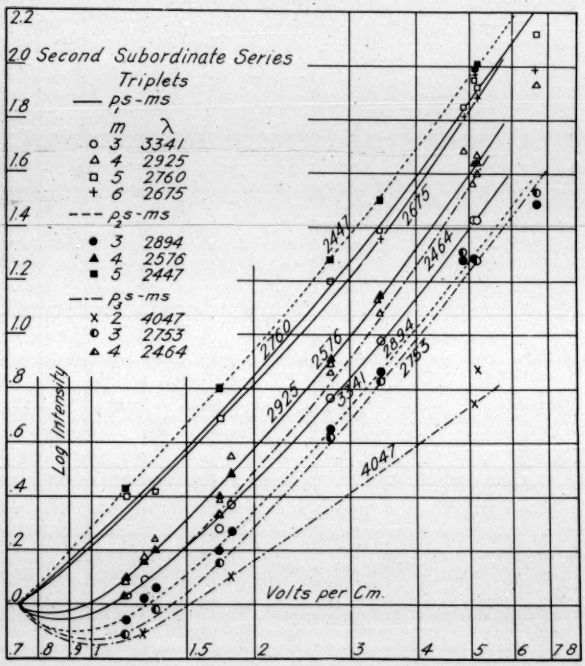


Variations of intensity of lines of the first subordinate series (triplet) of mercury with exciting potentials gradient in arc, p_2d_2, p_2d_3, p_3d_3 levels.

The data for the 29 lines measured are collected in the accompanying tables grouped according to series. In computing voltage gradients, 11 volts have been subtracted from the over-all voltage drop to allow for cathode drop. The gradient of .75 volt per cm. corresponds to an over-all voltage drop of 23 volts, the lowest voltage at which the arc

could be maintained. The logarithmic intensity at this gradient is taken arbitrarily as zero for every line. The temperatures given are those recorded by the thermocouple outside the tube and are only rough approximations to the

FIG. 3.

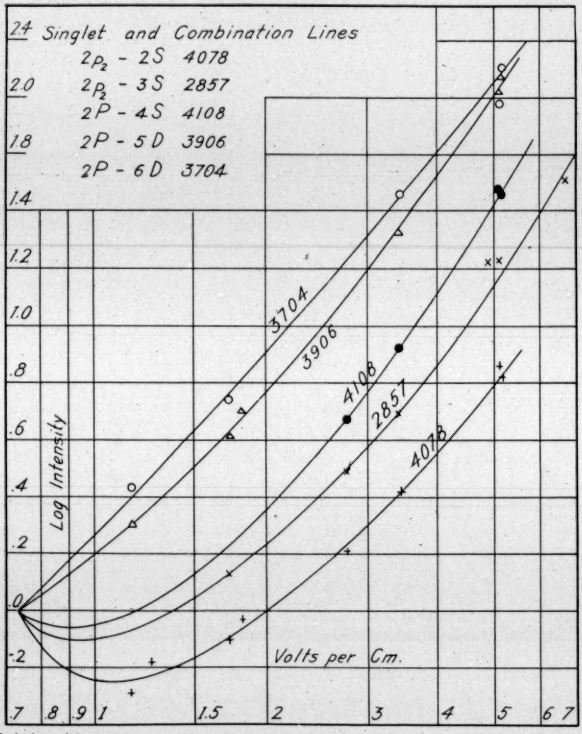


Variation of intensity of lines of the second subordinate series (triplet) of mercury with exciting potential gradient in arc.

temperature of the vapor. The pressures are those of saturated mercury vapor at the recorded temperatures. In order to indicate the comparative brightness of the lines, densities have been given for the spectrum taken with the stop of 5.26 sq.cm. area at .75 volt per cm.

In the graphs of these data logarithmic intensities have been plotted on a linear scale against voltage gradients on a logarithmic scale, so that a straight line graph indicates an

FIG. 4.



Variation of intensity of miscellaneous lines of the mercury spectrum with exciting potentila gradient in arc.

energy emission proportional to some power of the voltage gradient.

From the tables, and more obviously from the graphs, several generalizations may be drawn. Above a gradient of

1.5 volts per cm. all the curves tend to become parallel. This is the graphical indication that the brightnesses of the lines relative to one another vary but little at exciting voltages above 1.5 volts per cm., but that all increase in the same ratio. This behavior is apparently due to practically all of the vapor being completely excited or ionized above this voltage. The increase in brightness should then be proportional to the amount of vapor in the tube. This is roughly verified if the vapor pressure is assumed to be saturated. The energy radiated by each line in this region is proportional to about the 2.3 power of the voltage gradient.

Below 1.5 volts per cm. there is to be observed very considerable variation in the relative brightnesses of the lines. Considering the lines of a series there is seen to be a systematic variation in the voltage intensity curves as the term number, m, is varied. The lower members increase in brightness very slowly and may even decrease to a minimum of intensity below 1.5 volts per cm. As the term number rises the lines increase in brightness with voltage more and more rapidly, that is, the curves are more and more steep. (The apparent exception of the line 2675 to this generalization may be accounted for as due to error of observation. A small change in the data no greater than the probable error would bring it into concordance.) The separation between the curves of one member and that of the succeeding member grows steadily less with increasing term number, so that we may presume that the curves approach a limiting curve which would correspond to the limiting member of the series. In the second subordinate series and in the group of $2p$ and $2P$ lines, the curve of the last lines observed has throughout the constant slope which all the lines have above 1.5 volts per cm. But in the first subordinate series the curves of the higher members rise more rapidly at the lower voltages than at the higher.

Curves of the same triplet lie quite close together; in some cases they are identical within the precision of measurement, notably 3126 and 2967, 2894 and 2753, 2925 and 2576; but, on the other hand, others, though close, are yet distinctly different, as 3341 and 2894, 2803 and 2482. Athanasiu reported that 2652, 2967, and 3026 all maintained the same

relative brightness under conditions quite similar to these maintained in the present experiments. This is in disagreement with the present results.

This work was proposed by Professor C. B. Bazzoni and the author is indebted to him for assistance in its prosecution. Acknowledgment is also due to Professor Jas. Barnes and the authorities of Bryn Mawr College for the use of the densitometer.

UNIVERSITY OF PENNSYLVANIA,
PHILADELPHIA, PA.,
November 1928.

COMMITTEE ON SCIENCE AND THE ARTS.

ABSTRACTS OF REPORTS.

PERMALLOY.

HALL OF THE INSTITUTE,
PHILADELPHIA, December 7, 1927.

No. 2876.

The Franklin Institute of the State of Pennsylvania, acting through its Committee on Science and the Arts, investigating the invention of Permalloy, by Mr. Gustaf W. Elmen, of New York, reports as follows:

The name Permalloy is the trade name applied to certain nickel-iron alloys that, when properly heat-treated, possess certain magnetic properties to an unusual degree. In order that these properties may be developed these alloys must contain more than thirty per cent. of nickel and have the crystal structure of nickel instead of that of iron.

While the desired qualities of high permeability, ease of magnetization under weak magnetic fields and small hysteresis loss, are found whenever the nickel is over 30 per cent., these are found in the highest degree in permalloy containing about 80 per cent. of nickel. The following composition was found by analysis in a bar made by the inventor:

Ni.....	78.23 per cent.	Cu.....	0.10 per cent.
Fe.....	21.35 per cent.	C.....	0.04 per cent.
Co.....	0.37 per cent.	S.....	0.035 per cent.
Mn.....	0.22 per cent.	Si.....	0.03 per cent.
		P.....	Trace

Carbon is the most harmful of the elements present but the heat treatment given to the alloy to secure the highest permeability overcomes its effect.

This material was developed in the laboratories of the Bell Telephone Company, through the investigations of Gustaf W. Elmen to whom a United States Patent No.

1,586,884 was granted on June 1, 1926 on an application filed May 31, 1921.

One of the objects of the invention as stated in the patent is to provide a suitable loading material for signaling conductors to increase their range and speed of operation. Another object is so to apply this material to a conductive core as to produce a highly efficient transmission line for long range, high speed signaling.

In order to prevent the development of eddy currents due to the rapid changes in the magnetizing forces to which the material is subjected, it is used in a laminated form and its composition is such as to provide high resistivity.

In telegraphic and telephonic transmission it has been the practice to increase the inductance of the circuits by winding the conductors in series turns over an iron core in what are called loading coils.

Many years ago, Heaviside investigated the transmission of signals and acertained that the best results, i.e. highest speed and clearest signals, were obtained when the conductor approached, in its characteristics, as closely as possible a pure resistance.

In most conductor systems this requires the neutralization of the capacity by the addition of inductance.

Pupin developed this in the form of lumped inductance units, or loading coils, but since the capacity to be neutralized is distributed it is desirable that the inductance should be distributed also.

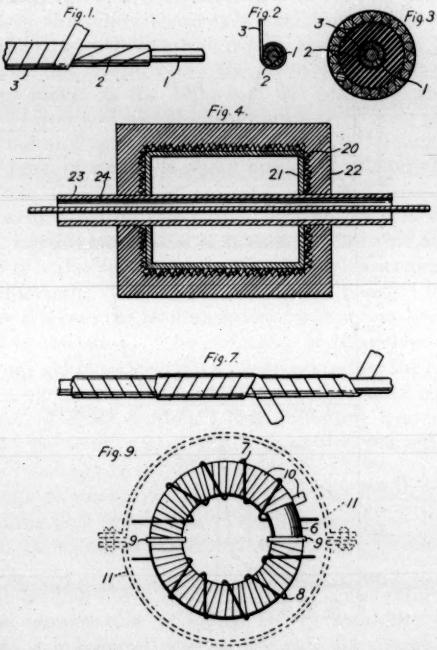
The main difficulty in applying these methods of improvement has been that there was no material known having sufficiently high permeability to make the methods effective.

In the manufacture of permalloy, iron and nickel of a high degree of purity in the proportion of about 21½ per cent. of iron and 78½ per cent. of nickel are fused together in an induction furnace. It is then poured into a mold and cooled. The resulting bar is then repeatedly swaged until it is changed into a rod. This is then drawn through a die until it becomes a wire of size about B. and S. No. 20. This wire is passed through several pairs of rolls and flattened into a tape about 0.006 inch thick and 0.125 inch wide.

It is then passed between cutting rolls that trim its edges squarely and give to the tape an exact width.

Figure 1 of Plate I shows how this permalloy tape is applied to the conductor. This conductor is composed of a bundle of copper wires, *1* and *2*, the combined area being equivalent to size No. 5 on the B. and S. gauge. Six special shaped strands, *2*, are wound helically around a central con-

PLATE I

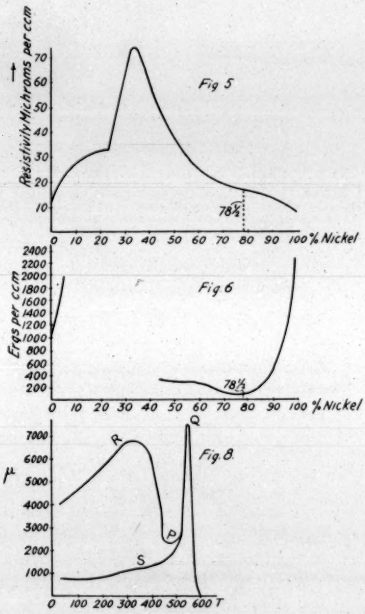


ductor, *1*, fitting it closely and leaving a smooth surface. Around this is wound the permalloy tape, *3*, as shown, care being taken to fit the tapes without overlapping.

The taped conductor is then drawn through an electric muffle furnace, Figure 4, having the heating elements, *20*, between the fire clay muffle, *21*, and the fire brick *22*. The

conductor passes through a copper lined iron tube, *24*, which extends through the middle of the furnace and projects several inches beyond the furnace walls. The conductor is drawn out from the furnace in a straight line to such a distance that it is cooled to nearly room temperature before it is

PLATE II



wound on a reel which should be of large diameter to prevent injuring the conductor, thus affecting its permeability.

After the heat treatment the loaded conductor, if designed for a submarine cable, is insulated, armored and mechanically reinforced by being wrapped with a layer of gutta percha, jute and finally a layer of helically wound steel wires as shown in a cross section, Figure 3.

Figure 9 shows the method of constructing a loading coil. Small permalloy wire is insulated and wound into a core *6*, and the windings *7* and *8* are mounted on this core after it has been wound with insulating tape, *10*. In order to secure constant permeability the core is sometimes provided with gaps filled with non-magnetic material as at *9*.

Experimentation has shown that with varying proportions of nickel in this alloy there is a variation in its magnetic characteristics and these are shown by the curves in Plate II. Figure 5 of this Plate shows the relation between the percentage of nickel in the alloy and the resulting resistivity. An alloy of iron and nickel has a higher resistivity than that of either metal alone and the resistivity of Permalloy, having 78½ per cent. of nickel, is about one third greater than that of pure iron.

It is evident, from this curve, that an alloy of about 34 per cent. has the maximum of resistivity and this alloy may be found of value for some uses in which the permeability is not so important.

Figure 6 shows the hysteresis loss for different percentages of nickel in the alloy. The ordinates of this curve give the work in ergs per cubic centimeter represented by the hysteresis loop for a maximum induction of 5000 c.g.s. units per square centimeter of cross section. The minimum hysteresis loss is at 78½ per cent. nickel and it is at this percentage that the alloy has its maximum permeability.

The curves shown in Figure 8, show the relation between temperature and permeability when a specimen having 70 per cent. of nickel is subjected to a magnetizing force of $H = 0.03$.

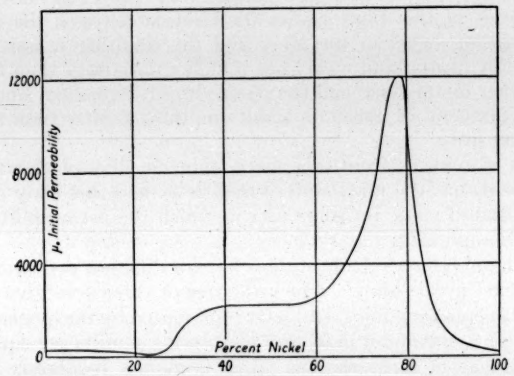
If the cooling down from its critical temperature, that is from the temperature at which the permeability suddenly falls from a maximum, was at the rate for giving the best results, the curve will be of the form Q, P, R, showing a rapid fall from Q to P and then rising to R. If, however, the cooling was too fast or too slow, the curve may be like Q, P, S, with no recovery of permeability.

Thirty-three claims are allowed in this patent of which the eighth and tenth are as follows:

"A magnetic material characterized by a higher permeability than that of iron at magnetizing forces of 2/10 of a gauss or less and lower hysteresis loss than iron, comprising at least nickel and iron of the magnetic group, the nickel being within a few per cent. of 78½ per cent. of the nickel-iron content."

"The method of developing high permeability at low magnetizing forces in a magnetic material which comprises annealing said material, bringing it to a temperature approximating its magnetic transition temperature, and then cooling at a rate intermediate an annealing rate and a rate at which undue stresses and strains would be set up therein."

PLATE III



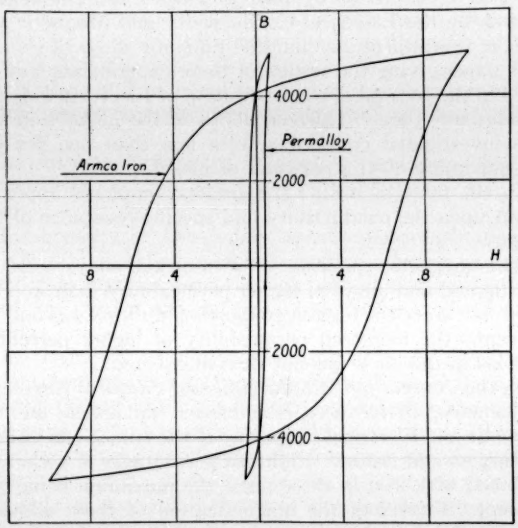
A comparison of the curve in Plate III with Figure 6 of Plate II shows the correctness of the statement that the maximum initial permeability occurs at the same percentage of nickel as the minimum hysteresis loss, at 78½ per cent. This plate is from a figure given in a paper on "Permalloy" by Mr. Elmen, printed in the JOURNAL OF THE FRANKLIN INSTITUTE for May, 1923.

Another figure in this paper, Plate IV, shows two hysteresis curves in which the magnetization is carried to a maximum of 5,000 gausses. In the loop shown, the magnetizing force necessary to obtain a certain magnetic density is less in permalloy than in iron and the hysteresis loss is much less than that in the iron.

One of the most successful applications of Permalloy has been in the loading of submarine telegraph cables for the purpose of increasing the speed of transmission.

An early attempt to obtain increased speed was made with the Key West–Havana telephone cable which is loaded with soft iron wire having a diameter of 0.008 inch, the permeability of which was the greatest of any material that could be obtained at the time this line was built.

PLATE IV



The New York–Azores cable, a Permalloy loaded line, has twenty times the permeability of the Key West–Havana line and has an inductance of about 54 millihenries per nautical mile.

This cable has been in operation for about three years and with the improvements that have been made in the terminal apparatus has attained a speed of 1900 letters per minute, about four times the speed of an unloaded cable of the same size and length.

The increasing use of devices and instruments in which magnetism was applied led to an extended study of magnetic

materials. The methods employed in these studies and the results obtained were fully discussed in a book on "Magnetic Induction in Iron and Other Metals" by J. A. Ewing, published in 1894. This book contained essentially all that was known regarding the prior art at the time of its publication.

Subsequent to the publication of this book many experiments were carried on by Barrett, Brown and Hadfield in a research on the Electrical Conductivity and Magnetic Properties of upwards of one hundred different alloys of iron.

A paper giving the results of these experiments was published in the Journal of the Institution of Electrical Engineers of February, 1902. Thirteen alloys of the nickel-iron series were investigated containing from less than one per cent. to a maximum of 31.4 per cent. of nickel.

In the table of results the greatest emphasis appears to be put upon the conductivity and specific resistance of these alloys.

The magnetic qualities of iron-nickel alloys were also investigated and alloys of higher permeability than iron were found, but since the highest proportion of nickel was only 31.4 per cent., the increased permeability of higher percentages of nickel in the alloy was not then discovered.

In the *Journal for Metallurgical and Chemical Engineering* for January, 1910, there is published an article on "The Magnetic and Electrical Properties of the Iron-Nickel Alloys" by Burgess and Aston. Eighteen percentages of nickel were combined with iron in these tests, the maximum being 75.06 per cent. Curves of the magnetization of these alloys are shown both when the alloy is unannealed and when annealed at 675° and 1000° C. Hysteresis loops and electrical resistance curves are also shown. This paper gives quite full results of the investigation over the range covered but it does not carry the research far enough to compare with the 78½ per cent. of nickel in the permalloy composition.

The use of Permalloy in the field of electrical communication is constantly increasing.

It is now being used as a core in loading coils. For this purpose the Permalloy is reduced to a finely divided state by grinding and after a suitable heat treatment it is subjected to a pressure of 200,000 pounds per square inch between steel

dies resulting in the present powdered-permalloy core. This core has only one third the volume of the iron core previously used thus requiring much less space in the loading vaults. These loading coils have greater magnetic stability and permeability than with the iron cores previously used and are being placed in the telephone plants of the Bell Telephone Company at the rate of about 700,000 each year.

Another use for Permalloy is in the audio frequency transformers of radio receiving sets in which the highest quality of reproduction is desired.

It is also used in telephone and telegraph relays which are now being manufactured at the rate of about 20,000 each year.

The inventor submitted a piece of apparatus for the demonstration of the relative permeability of iron and permalloy.

This consists of two small transformers, each supplied with a primary winding of eight turns and a source of electric current that can be sent through either of these coils as desired, the core of one coil being iron and that of the other permalloy.

The secondary windings, consisting of many turns, may each be connected to a galvanometer, the throw of the needle indicating the amount of the induced voltage when the current in the primary coil is reversed. Although there are many more turns on the secondary of the iron core than on that of the permalloy core, a reversal of the current on that core shows only a very small throw of the galvanometer needle while a reversal on the coil with the permalloy core gives a throw of more than half the entire scale reading.

In consideration of his extended researches in the magnetic characteristics of nickel-iron alloys resulting in the invention of Permalloy, an alloy having high permeability and low hysteresis loss, and of its successful uses for the loading of submarine telegraph cables to secure high speed transmission, for the cores of loading coils and audio frequency transformers, THE FRANKLIN INSTITUTE awards its ELLIOTT CRESSON MEDAL to MR. GUSTAF W. ELMEN, of the Bell Telephone Laboratories, New York.

PRECISION REFRACTOMETER.

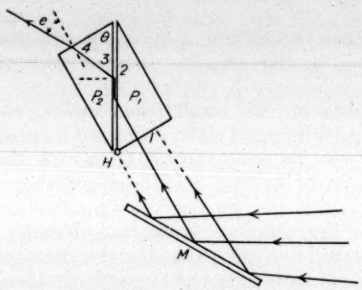
HALL OF THE INSTITUTE,
PHILADELPHIA, April 4, 1928.

No. 2877.

The Franklin Institute of the State of Pennsylvania acting through its Committee on Science and the Arts, investigating the Precision Refractometer invented by Mr. Warren P. Valentine, of Philadelphia, Pennsylvania, reports as follows:

When rays of light fall upon a prism of a transparent substance and are transmitted by it their paths are deviated

FIG. 1.



by an amount depending on the angle of the prism and on the value of its refractive index. This deviation varies also with the wave-length of the light used.

The most accurate determination of indices of refraction is made by the careful observation of the angle of deviation produced in various wave-lengths of light by an accurately worked prism of fairly large dimensions of the material, the index of which is to be determined. This method takes a considerable amount of time to carry out and is even more troublesome when applied to liquids as the liquid has to be contained in a hollow prism built up of accurately plane parallel pieces of glass.

There is a large amount of refractometry which has to be done in connection with various industries for which the highest accuracy is not demanded and for which the time

and trouble involved in such determinations would not be justified by the expense. For such purposes a number of instruments have been devised which are very rapid in use and require a minimum of material and labor. One of these is the Abbe refractometer. The essential parts of this instrument are shown in Figure 1.

P_1 and P_2 are two prisms of dense glass. Surfaces 1, 3 and 4 are polished, surface 2 is mat. The prism P_1 is hinged at H so that it can be swung away from P_2. In determining the index of a liquid, a drop is placed on surface 2 which is closed up until it is nearly in contact with surface 3, the liquid drop being squeezed out into a thin film. Monochromatic light from a suitable source is directed towards the prism system by a mirror M, it strikes the mat surface 2 and is scattered into the liquid film and the prism P_2. Since no ray can enter P_2 with a greater angle of refraction than that of the ray at grazing incidence, the emergent rays when collected by a telescope will all converge to points on one side of a line in the focal plane. The field will therefore be divided into a dark and a light portion with a sharp edge, upon which the cross-hairs in the telescope are set. The rotation of the telescope is registered on a circular arc which is graduated in units to suit the problem under investigation.

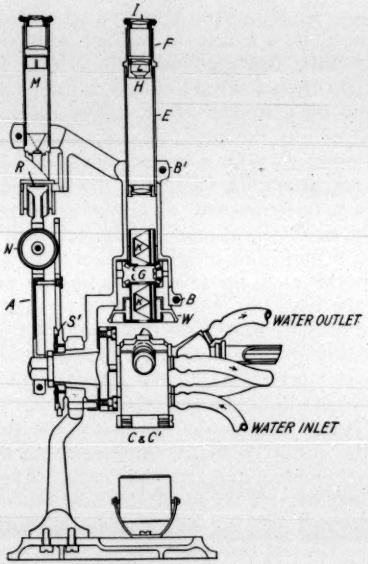
The instrument can also be used with white light by using two direct vision prisms in the telescope to compensate for the dispersion of the liquid and the prism P_2.

The Valentine refractometer which is the subject of this report, is the result of many improvements on the Abbe refractometer as manufactured by Zeiss. These improvements resulted from a study of this instrument when Mr. Valentine decided in 1917 to construct a duplicate of it. On account of the fact that mechanicians who were competent to do instrumental construction of high grade, were engaged in work for World War requirements and on account of the difficulty of obtaining the necessary materials, especially optical glass, this instrument was not finished until January, 1919. Since no improvements had been made in the Zeiss-Abbe instrument since it was originally designed, several desirable changes that would add to its stability and accuracy were considered and Mr. Valentine constructed a second

refractometer in which he incorporated changes in its mechanical and optical design that would secure these results.

Two models are referred to in the following description: the Precision Refractometer, with sector scale of refractive indices divided to the third decimal by estimating tenths of the smallest division; and the Improved Precision Refractometer, Figure 2, with sector scale divided to the fourth

FIG. 2.



decimal and reading to the fifth decimal by estimating tenths of the smallest division.

In order to avoid an error due to parallax, Mr. Valentine redesigned telescope *E*. The field lens *L* and the cross-hairs *H* are fixed relatively to each other. The eye-lens, *I*, being mounted in a screwed cell, may be adjusted to the sight of the particular individual observer and the entire ocular

focussed upon the border line. The entire telescope unit is self-contained and held rigidly in place by means of the clamp screw B' and thus may easily be removed for inspection or cleaning. For reading the extremely small divisions on the sector scale a small compound microscope is provided to which a drum scale is fitted for splitting the divisions into tenths.

Every possible means is employed to secure the highest accuracy. The refractive index of the glass of which the refractory prism is made is determined to the sixth decimal. Errors of flatness in the refracting and transmitting surfaces are held within one third of a wave-length.

The Valentine Refractometers have come into general use. They are used extensively in the paint, oil and drug industries, in chemical and physical laboratories, in dispensaries to measure the quantity of albumin or protein in blood serum and urine. They are also used to determine the oil content of flaxseed, cotton and other oil-bearing seeds, to control the melting point in the hardening of vegetable and mineral waxes and the hydrogenation of soap stock, to measure the total solids in tomato juice and catsup and the concentration of sugar syrups as in sugar manufacture and the sugar content of fruit jellies and preserves. Mr. Valentine submitted with his application a list of almost a hundred users of his refractometers.

In consideration of the meritorious work shown in the improvement of the mechanical and optical parts of the Abbe refractometer, thereby increasing its accuracy, THE FRANKLIN INSTITUTE awards its EDWARD LONGSTRETH MEDAL to MR. WARREN P. VALENTINE, of Philadelphia, Pennsylvania.

THE WORK OF ARTHUR GRAHAM GLASGOW.

HALL OF THE INSTITUTE,
PHILADELPHIA, March 7, 1928.

No. 2880.

The Franklin Institute of the State of Pennsylvania, acting through its Committee on Science and the Arts, investigating the work of Mr. Arthur Graham Glasgow in the Gas Industry, reports as follows:

Arthur Graham Glasgow was born in Buchanan, Virginia, May 30, 1865. He was graduated at Stevens Institute of Technology in 1885, with the degree of M.E. He was in the service of the United Gas Improvement Company from 1885 to 1891 in various capacities, afterwards becoming Engineer and General Manager of the Standard Gas Light Company of the City of New York. Since May, 1892, he has been managing partner in Europe of the firm of Humphreys and Glasgow, Ltd., contracting and constructing gas engineers. Under his direction, this firm has designed, constructed and brought into successful operation, carburetted gas water works throughout the United Kingdom, the Continent of Europe, and in Australia, New Zealand, China and other countries. He was President of the official delegation representing the United States Government at the World's Gas Congress at Paris in 1900. During the late war, Mr. Glasgow was Fixed Nitrogen Administrator in the United States War Department.

Mr. Glasgow is the author of numerous papers presented before various organizations in this country and in England. For his paper on "Carburetted Water Gas as a Coal-Gas Auxiliary," presented before the American Gas Light Association in 1899, Mr. Glasgow received the Beal Medal. The committee awarding this medal, said: "Mr. Glasgow treated his subject in a masterly manner. The paper is broad in its scope and of great and general usefulness to the gas engineer. The deductions are reached by the author after a thorough and analytical examination of all the underlying bases of gas manufacture."

Mr. Glasgow is a member of the Institution of Civil Engineers of Great Britain, Institution of Mechanical Engineers of Great Britain, The American Society of Mechanical Engineers, American Society of Civil Engineers, American Gas Association, and the Western Gas Association.

In its investigation of the work of Mr. Glasgow the Institute has been in communication with many leading authorities and executives in the gas industry who were practically unanimous in their appraisal of the high value of Mr. Glasgow's achievements and of his standing as an engineer and executive.

Mr. Glasgow's work may be summarized as follows:

1. As an outstanding authority on water gas manufacture, he has been associated with nearly all of the important improvements and developments that have been brought about in water gas technology and practice.

2. As the author of some thirty-five papers on various phases of gas manufacture, he has made important and lasting contributions to the literature of the general subject.

3. In the commercial development of water gas manufacture and utilization, he has played an important part, as shown by the fact that he has been granted twenty-six patents for various inventions of equipment and methods, and that to date about 1,885 water gas sets have been installed by his firm in various countries, with a total daily capacity of 1,870,900,000 cubic feet.

In consideration of his outstanding and valuable work in the technology and development of the manufactured gas industry, THE FRANKLIN INSTITUTE awards its WALTON CLARK MEDAL to MR. ARTHUR GRAHAM GLASGOW, of London, England.

Action of Light upon Ferric Citrate. According to JOSEPH B. BURT (*Jour. Am. Pharm. Asso.*, 1928, *17*, 650–658) the magma of ferric hydroxide does not dissolve completely in a solution of citric acid when the two compounds are present in exactly molecular proportions; an excess of the acid is required to produce complete solution of the hydroxide. Study was made of the action of sunlight upon the resulting solution, (1) as prepared, (2) after complete neutralization with ammonia water, and (3) after half neutralization with that base. At the end of two days, the solutions containing free citric acid had become colorless, and carbon dioxide gas had been evolved from them. Acetone was also formed; but formaldehyde was absent. One molecule of citric acid was oxidized with the formation of one molecule of acetone, three molecules of carbon dioxide, and one molecule of water. The acid solution as prepared yielded more gas than the half-neutralized solution. The completely neutralized solution became lighter in color, but did not become colorless; and no gas was evolved from it. The end point of the reaction apparently depended upon the complete reduction of the iron to the ferrous state, rather than on the exhaustion of an excess of citric acid. Use of quartz vessels instead of vessels of ordinary glass did not increase the yield of carbon dioxide, other conditions being constant; therefore, the light need not necessarily be rich in ultra-violet rays. The end point was not very definite, for the iron in the reduced solution was partially re-oxidized when this solution was permitted to stand in diffused light. It is recommended that solutions of this type, for example, the elixir of iron, quinine and strychnine, and the elixir of iron, quinine and strychnine phosphates, be rendered as nearly neutral as possible, and then be kept in containers of amber glass.

J. S. H.

Essential Oils as Antiferments. ELLERY H. HARVEY (*Am. J. Pharmacy*, 1928, *100*, 524–529) has determined the value of thirty-two typical essential oils by means of their influence on the hydrolysis of sugar by yeast. The essential oils of clove, cinnamon, thyme, and wintergreen were found to rank high in effectiveness. Absorption of ultra-violet radiation affected both the color and the antiferment value of essential oils. Such oils should be kept in brown bottles, and not in blue or clear glass bottles. Acetic acid, in concentrations above 1.5 per cent., was found to be an efficient antiferment.

J. S. H.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

DISPOSAL OF WASTE CRANK CASE OILS.

A REPORT, containing recommendations for controlling the disposal of waste automobile crank case oils so as to avoid contamination of public sewers, was issued under date of November 15, 1928, by J. B. Gordon, Sanitary Engineer of the District of Columbia.

This report contains excerpts from replies received to a questionnaire sent by Mr. Gordon to many cities in the United States and foreign countries. These replies show that a large percentage of municipalities recognize the danger involved in allowing great quantities of oil, often containing considerable amounts of gasoline, to enter the sewers. Many cities have adopted regulations covering the disposal of waste oils, but often such regulations are not successful in accomplishing the desired object, because, while the regulations prohibit the dumping of oils into the sewers, no recommendation is made as to what is to be done with them.

The report gives briefly the results of an investigation conducted by the Bureau of Standards, in which it was shown that by the use of a special rerefining process oils suitable for use in automobile engines can be recovered from waste crank case oil. A table gives the properties of the new, used, and rerefined oils, based on tests of samples obtained by the bureau from the Post Office Department and the Office of the Superintendent of Public Buildings and Parks. In a second table values are given for used and rerefined oils collected from a public filling station.

Mr. Gordon concludes his report as follows:

"With all the information as cited herein at hand, including the reports from the United States Bureau of Standards showing the value of the product, the Plumbing Regulations of the District of Columbia were recommended to be revised in this respect and amended to require the main-

* Communicated by the Director.

tenance by garages, filling and service stations, automobile repair shops, etc., of approved containers or underground tanks, separate and apart from garage traps, for the retention of all waste crank case oils, the contents of the same to be removed by the occupant or operator at regular intervals or upon notice from the District authorities. Regulations for the safe transportation of this commodity are also included in the proposed amendments.

"The adoption of these regulations will necessarily require provision to be made for the safe and systematic disposition of the waste oil so retained. The dumping grounds in the District and vicinity have prohibited the receiving of such refuse (waste oils or contents of garage traps) and in order that the public may be fully protected in the matter of disposition of these accumulations of waste oil retained under such regulations, particularly as to the more or less isolated sections being properly provided with disposal service, a proposal is being submitted for approval whereby a contract may be made under competitive bidding for the collection, transportation and disposition of such waste oil as may become available. Such services as may be contracted for will be guaranteed by proper bond, regulated by direct supervision of the District Government as to scope of service, maintenance of facilities for collection, transportation and disposition and all other conditions which will make for a thorough and systematic service.

"In view of the fact that the rerefined product has been determined usable and valuable, it is expected that this contract service will be without cost to either the District or the public, the contractor realizing his earnings from the ownership of the available oils so collected. No monopoly is to be granted the contractor except that requests made by the public on the District for the removal of oil will become an order on the contractor for the rendering of this service. In this way, the outlying sections of the District will be supplied with disposal service on a par with the more densely populated part of the city. The occupant or operator of any premises may dispose of the oil so retained in any manner he may choose the only stipulation being that its disposition must be in accordance with the Regulations.

"Under these conditions it is estimated that more than 200,000 gallons of waste oil should be available for collection annually. It is believed that the public will be quick to realize the importance attaching to the exclusion of this waste from the public sewers as well as appreciate the advantage to be gained from its free and regular removal, especially the latter as it costs the operator some 2¢ to 3¢ per gallon to have it removed when deposited with silt in garage intercepting traps. A decided improvement of the condition of the interior of public sewers may confidently be expected to result from the adoption of these measures, the danger from fires and explosions in sewers minimized, thousands of gallons of oil which has heretofore been a source of danger and expense conserved for reuse and a commercial enterprise of appreciable moment encouraged and developed."

MELTING POINT OF NICKEL.

THE bureau has practically completed a determination of the melting point of nickel, based upon two samples prepared in its own laboratories. These samples, known as N19 and N23 were from different melts, and each was found to contain about 0.06 per cent. of impurities, chiefly iron, cobalt, and copper. The main difference between the two samples lies in the fact that N19 contains these impurities in about equal parts, while N23 contains more iron and is nearly free of copper.

Nine freezes were observed on N19 by two observers. The average value obtained was 1454.9° C., the maximum deviation of any one freeze being 0.5° C. In addition, the readings of each observer, taken alone, average 1454.9° C. The previous value which has been generally accepted as the best is 1452° C.

Eight freezes have been taken on the second sample, N23, the average being likewise 1454.9° C., and the average of each observer's readings, taken alone, again being 1454.9° C., the maximum deviation of any reading of either observer being 0.4° C.

The bureau believes that enough work has been done to show conclusively that the two samples of nickel are so

nearly alike that all the observations on the 17 freezes may be treated as observations of the same sample. Considered from this point of view, 17 readings have been secured with a maximum deviation of 0.5° C., and a probable error of 0.05° C. It is believed that the result secured is not in error by more than 1° C. on the International Scale. This conclusion is based on the following estimates of the errors that may be present:

Source of Error.	Amount. Degrees C.
Purity of nickel	− 0.5
Transmission of sector	± 0.1
Effective wave-length	± 0.5
Setting pyrometer	± 0.2
Maximum	1.3

THE PRODUCTION OF XYLOSE ON A SEMI-COMMERCIAL SCALE.

THE Alabama Polytechnic Institute and the University of Alabama, coöperating through the Alabama Industrial Development Board, have arranged to work with the Bureau of Standards in the erection and operation of a small factory in which experiments can be made on the recovery of xylose from cotton-seed bran and peanut shells, on a semi-commercial scale. The plant is now under construction by the Federal Phosphorus Company at Anniston, Alabama.

Xylose at present is a rare sugar, selling for about $100.00 per pound, but possessing properties which ought to make it a desirable material for use in the food, textile, and leather industries. Its investigation is part of an extensive research program which the bureau is conducting with the object of discovering new uses for the waste products of agriculture. This work does not stop with the production of a marketable material in the laboratory, but includes the discovery of profitable ways to dispose of this material. As a matter of fact, it is not so difficult to find how to make things from these waste products. The difficulty lies in finding how to make money out of them. The chief difficulties are economic rather than chemical.

Therefore, the bureau has found it necessary to build small factories in which the laboratory processes can be tried out

on a production basis, thus making it possible to figure manufacturing costs with some degree of assurance. These small plants are useful in another way as well. They make possible the production of enough material to be actually used by likely consumers in their own plants, so that markets for the product may be explored. Thus, in the case of xylose, it seems probable that it will find a ready market in the industries mentioned, but there is no way to answer this question definitely while it is a laboratory curiosity. The erection of the plant at Anniston, with a capacity of 100 pounds of xylose per day with a manufacturing cost probably not exceeding fifty cents per pound, should soon give definite information on the subject.

USE OF ANTIOXIDANTS IN PRESERVING RUBBER.

DURING the past two or three years several substances have been placed on the market which are designed to be incorporated in rubber compounds to prevent or at least retard deterioration through aging. These substances are designated by the general name of antioxidants. In the case of one or two of these, it has been claimed that it is beneficial to dip rubber articles already vulcanized into solutions of the antioxidants. In order to obtain data on the influence of this dipping process on the aging of rubber articles, tests have been made with five different antioxidants. Inasmuch as the penetration of antioxidants during the dipping process is probably not very great, the logical products on which to test the effect are thin rubber products such as surgeons' gloves, rubber bandages, rubber bands, etc. Numerous samples of these articles were dipped in benzol-alcohol solutions of antioxidants for five minutes and then allowed to dry in the air. These were then exposed to aging under two conditions; first, indirect light such as in an ordinary room; and second, stored in the dark.

The results of tests of samples exposed to light show that in all cases the antioxidants have had a decided beneficial effect. Aging in storage is a much slower process than when rubber is exposed to light and to date no difference is apparent between dipped and undipped articles.

PROTECTIVE COATINGS FOR PAPER CURRENCY.

THROUGH the use of paper having increased fiber strength, the proportion of the United States paper money that is redeemed because of its being broken or torn has been reduced until it is almost negligible. Most of the paper currency that is now withdrawn from circulation on account of being considered unserviceable is declared unfit because of oil or grease stains, dirt, creasing, and other evidences of surface wear. On account of this condition, the coöperative investigation of the Bureau of Standards and the Bureau of Engraving and Printing has been extended to include a study of various materials for protecting the surfaces of the notes for the purpose of further prolonging the working life of paper currency.

At present, sheets of notes are resized with animal glue, but the demands of large scale production have prevented the employment of the low temperature drying methods that are essential for preserving in full the improvements contributed by the glue. The addition of tanning materials such as formaldehyde and of softening agents such as glycerine to the freshly sized paper has been studied, and the use of such reagents seems to offer considerable promise of improving the resistance of the resized notes to surface wear. The necessity of removing quickly the relative amount of water that is retained by papers resized with glue or other proteins such as casein or albumen, has caused attention to be centered quite largely on the use of protective materials that may be applied with organic solvents. It is planned, however, to study the present resizing equipment in order to determine if air-drying methods may be applied in order to enhance the effects of the glue sizing.

Various lacquers and varnishes have also been included in the investigation. Both cellulose nitrate and cellulose acetate lacquers have been tested. The work with varnishes has been limited in scope, but the materials used tended to make the paper surfaces somewhat sticky when warm and to increase the brittleness of the treated notes. Several nitrocellulose lacquers were tried but these were generally considered to be impractical for large scale application because of the strong and persistent odors of the solvents which were of the type of

amyl or butyl acetate, fusel oil, or similar materials. Objection was also raised to the excessively glossy surfaces imparted to the bills by the nitrocellulose lacquers used in the small-scale work.

With the coöperation of manufacturers of the material, the possibility of using cellulose acetate as the protective material has been studied quite extensively. Many combinations of different cellulose acetates, plasticizers, and solvents were tested until there was developed a solution capable of imparting to printed surfaces improved resistance to oil, water, and wetrubbing, without noticeably decreasing the high folding strength of the paper. The cellulose acetate forms a closely attached, invisible film that is free from gloss or stickiness and that does not bloom or discolor when dried at room temperature. Practical use of such a material would require special equipment for its application and for the removal of the organic solvents employed. In addition, as is the case with other surface protective agents, the question of the adherence of the overprinted sealing and numbering inks to such treated surfaces would require further careful study. These factors are now being considered.

Most of this work has been on a laboratory scale. It is now planned to undertake full-scale work with especial references to the drying of paper currency that has been resized with glue.

TESTS OF TERRA COTTA FACED WALLS.

THE Bureau of Standards in coöperation with the National Terra Cotta Society has just completed a second series of tests on terra cotta faced walls. The purpose of this work is to determine the best method of setting terra cotta ashlar on buildings. Owing largely to lack of information on which to base requirements, most of the building codes do not give specific details covering this subject.

The test walls were $4\frac{1}{2}$ feet wide and $7\frac{1}{2}$ feet high, and included the different types of construction now in vogue in different parts of the country. In some walls the terra cotta voids were filled with brick and mortar; in others the voids were partially filled with concrete grout; and in still others the voids were left unfilled. The strengths of these walls

were determined under different types of loading. On some walls, the load was applied on the backing wall behind the terra cotta; on others it was applied on the terra cotta facing only; and on still others it was applied uniformly over the terra cotta facing and the backing wall. The vertical deformations of the terra cotta facing and the backing walls were measured at different loads. The separation of the terra cotta facing from the backing wall was also measured during loading.

It appears from the results that the filled terra cotta facing adds considerably to the strength of the walls. These would apparently justify reckoning filled terra cotta as a portion of the thickness of walls. Such a possibility has decided commercial advantages in that the structural framework would not have to carry so great a load and, therefore, could be constructed more economically. The adoption of thinner walls would also permit the use of a larger floor area for renting purposes.

It was also the purpose of this work to compare the strength of walls containing terra cotta ashlar with two vertical partitions and six cells, with those walls containing ashlar having three vertical partitions and eight cells. Although it is possible to build brick filled walls with six cell ashlar which are equally as strong as those with eight cell ashlar, it was found that the concrete walls containing eight cell ashlar were stronger than those containing six cell ashlar.

DRYING BEHAVIOR OF SHALES AND CLAYS.

ONE of the investigations being conducted at the Columbus branch of the bureau is concerned with the drying behavior of several shales and clays of the glacial and alluvial type. A humidity dryer of semi-commercial size is used in this work. Water loss, shrinkage, and surface and center temperatures are determined on three green brick of commercial dimensions, the brick being made with a small combination pug mill and brick machine.

Although the data are as yet not sufficient to warrant drawing positive conclusions, a number of interesting facts have developed. It was found that a commercial sized brick

made from the Rutland (Ohio) alluvial clay could be made to lose about 76 per cent. of the added tempering water when dried at an average temperature of 26° C. and an average humidity of 94 per cent. for 244 hours. At this point an equilibrium appeared to be established. At no time was the temperature above 27° C. or the humidity below 92 per cent. The brick lost weight steadily at a temperature of 25.6° C. and a humidity of 97 per cent., even after it had already lost about 33 per cent. of the added tempering water.

The above is of interest in view of the fact that there is a rather prevalent opinion that clay ware will not dry out at such high humidities at this temperature range but on the contrary will absorb water.

Maintaining the same temperature, the humidity was lowered to 85 per cent. and the brick again lost weight. After it had lost about one ounce, the humidity was raised to 94 per cent. and the brick gained 0.5 ounce in weight and expanded 0.003 inch in 10 hours. The brick were dried at 105° C. for 15 hours and again subjected to a humidity of 94 per cent. at a temperature of 26.7° C. Under this treatment the brick absorbed water to the extent of 3 ounces (about 4 per cent. of the dry weight) and expanded 0.011 inch in 69 hours. Thus, after about 75 per cent. of the tempering water has been evolved, the clay can be made to gain or lose water at will by varying the humidity or temperature. However, the clay only regained about 50 per cent. as much water at 94 per cent. humidity after being dried at 105° C. as it held when equilibrium had been attained at this humidity after the drying process had been carried on for about 244 hours.

Two facts of importance are to be noted here: first, the reabsorption of water on proper change of condition, and secondly, the expansion which takes place coincidently. It is entirely possible that considerable dryer trouble might be traced to such change of conditions.

Shrinkage had practically ceased when about 75 per cent. of the tempering water had been eliminated, this being the time that the equilibrium had been attained apparently at 94 per cent. humidity. That is, this percentage possibly represents water present in a purely mechanical fashion. But, inasmuch as the remaining 25 per cent. of water apparently

exhibits equilibrium conditions at various humidities (temperature constant at about 26° C.), it is possible that this portion of the water is present in a definite adsorbed or hydrate condition, each of the hydrates having its characteristic vapor pressure.

A further interesting fact is that the shrinkage varies with the length of time required to dry the brick to the end of the shrinkage period. In the table below, it is to be noted that the shrinkage of the Rutland alluvial clay is in every case greater, the greater the length of the drying time.

Drying Time, Hours.	Total Shrinkage, Inches.	Shrinkage, Per Cent.
4	0.435	5.15
6	.456	5.42
8	.458	5.44
10	.469	5.56
12	.485	5.75
220	.522	6.23

From this it would appear that to state shrinkage correctly, the conditions under which the drying takes place should also be stated.

REACTION OF WATER ON THE CALCIUM ALUMINATES.

AN item on this subject was published in Technical News Bulletin No. 124 (August, 1927). A full report, giving in detail the results of this investigation, will appear in the December, 1928 issue of the Bureau of Standards Journal of Research, as Research Paper No. 34 under the title. "Reactions of Water on the Calcium Aluminates."

In this investigation the four calcium aluminates ($CaO.Al_2O_3$, $3CaO.5Al_2O_3$, $5CaO.3Al_2O_3$, $3CaO.Al_2O_3$) were made, and the mechanism of their reaction with water was studied. Tricalcium aluminate, which appears to be the only aluminate present in Portland cement of normal composition and normal properties, reacts so rapidly with water that the changes in composition of resulting solutions could not be followed. The monocalcium aluminate and the 3 : 5 calcium aluminate occur in cements characterized by a high alumina content. These two aluminates together with the 5 : 3 calcium aluminate and a high alumina cement react with

water to form metastable and supersaturated monocalcium aluminate solutions in the early periods. The metastable solutions decompose as the reaction proceeds with the precipitation of varying amounts of hydrated alumina and crystalline hydrated tricalcium aluminates with attendant increases in both the pH and molar ratio CaO/Al_2O_3, in the resultant solutions. If the metastable solutions are filtered from the reaction mixtures, they also decompose with the precipitation of the above phases. The resultant solutions, which are in equilibrium with these two phases, apparently attain the same composition as those left in contact with the calcium aluminates or high alumina cement for long periods.

A study of the subsequent changes in the aluminate solutions attended by increasing concentrations of calcium hydroxide, a product of the hydrolysis of the calcium silicates, shows that the alumina is almost completely precipitated from solution at a pH value of 12.0 or above.

The constitution of the aluminate solutions is discussed. Calculations based upon electrometric measurements and chemical analyses indicate that the alumina in the aluminate solutions is in combination with lime as the calcium salt of monobasic aluminic acid.

Tricalcium silicate and beta dicalcium silicate appear to hydrolize less in an aluminate solution than in water.

An electrometric titration study of aluminum chloride and calcium hydroxide is described. The conditions under which the aluminum precipitates as the hydroxide and as the calcium chloraluminate were studied. The formation of calcium chloraluminates from the reaction of calcium chloride with aluminate solutions is also described.

The optical properties of the various hydration products are given.

TEST OF SLATER'S CEMENT.

FREQUENT calls for tests on slater's cements have led to an improvised method for determining the adhesive and cohesive properties of such materials under conditions simulating those of service. The intended purpose of slater's cement is to produce and maintain a waterproof joint between slate shingles on the roof. The material is called upon to withstand

rigorous weather conditions and remain sufficiently plastic to accommodate itself to structural movements.

Previous tests have been confined mainly to determining the effects of high and low temperatures on the physical state of the cement. The new test was designed to determine if structural movements of a given magnitude will break the joint and permit leakage.

The test is modeled on a procedure now in use for testing "elastic pointing materials." A joint is made of two blocks of stone, one of which is fixed and the other so arranged that it can be moved toward or away from the fixed stone by means of a thread and nut. The movements can be accurately controlled and measured by a gauge between the two blocks. A cavity is made in the ends of the two blocks in such a way that it will hold water when the joint is sealed with the material to be tested. In testing a slater's cement, strips of slate shingles are sealed to the ends of the stone with sealing wax and then the joint is made between the slates with the cement in question. After the joint has set for one month the cavity is filled with water. If no leaks are noted the joint is stretched a small amount to simulate the effect of a structural movement.

Experiments so far completed indicate that a joint of slater's cement is usually capable of withstanding a movement of 0.01 inch at 70° F. after a drying period of one month, and some tests have indicated considerably higher values. This procedure seems to offer a satisfactory means of comparing different cements of this nature and to afford a basis for improvement of the product.

EFFECT OF WORKING GYPSUM MIXES AFTER STIFFENING.

CALCINED gypsum for certain purposes, such as for use in construction, must have a fairly quick set when mixed with water. However, sufficient time must be allowed to permit thorough mixing of the material and pouring into the forms before it starts to set. It has been generally accepted that the gypsum is useless if set has started before it is in place, inasmuch as little strength can be expected from the gypsum when worked through the set. Preceding the set a stiffening of the calcined gypsum putty occurs, and the question arose

as to whether the material could be used after the stiffening of the putty had started. Some tests have been made to determine for what period the putty can be worked without materially reducing the strength of the set gypsum.

A large batch of calcined gypsum was mixed with sufficient water to bring it to standard consistency, and three 3 by 6 inch cylinders were cast two minutes after mixing, and at one minute intervals thereafter until the paste was too stiff to be forced into the mold, even by using considerable pressure. A typical set of results is given below.

Time after Adding Water, Minutes.	Compressive Strength, lbs./in.²
2	1,070
3	1,060
4	1,100
5	1,070
6	910
7	710

The material for the specimens at six and seven minutes had to be pressed into the mold. Similar results have been obtained using retarded mixes with longer time intervals between the casting of the specimens, and using mixes containing various proportions of wood chips. It seems evident, therefore, that no weakening of the set gypsum occurs when the paste may be molded or formed without exerting undue pressure.

LIGHT-COLORED GROUND COATS FOR SHEET IRON.

A TREATMENT of the metal used in this work has been described in Technical News Bulletin No. 133 (May, 1928). The metal is coated with a film of cobalt nitrate. This film decomposes at 300° C. (about 570° F.) and is thus rendered insoluble. The light-colored enamel is then applied to the treated metal.

Sixty-three light colored ground coat enamels have been prepared and tested, the majority of them being clear glasses, although several of the frits [1] were very white and opaque. These were divided into four groups as follows:

[1] A term used to denote the glass which forms the basis of a vitreous enamel.

Ingredient.	Group.			
	G-2.	G-3.	G-4.	G-5.
	Per cent.	Per cent.	Per cent.	Per cent.
Feldspar	—	—	—	42
Silica	65	65	60	16.7
Boric acid	5 to 17	5	5	11.3
Sodium oxide	12 to 27	10 to 19	12 to 21	13.5
Lead oxide	0 to 18	6 to 15	6 to 18	—
Fluorspar	—	0 to 9	5 to 17	1.5 to 11.5
Cryolite	—	—	—	0 to 10
Sodium antimonate	—	—	—	0 to 10

When applied to treated metal, the composition of the enamel does not seem to be the most important factor in its adherence. The strength of the test specimens and the type of fracture obtained depend more upon the thermal expansivity of the enamel and the firing temperature. It appears, therefore, that any ground coat enamel now in use may be converted into a light-colored one by simply eliminating the colored oxides and giving the metal a suitable treatment such as described above.

Of the four groups in the above table, G–2 contained the best enamels as these did not have as much tendency to crawl or blister, and had a lighter color than any of the others.

If the Group G–2, containing 65 per cent. silica, be plotted as the ternary diagram $PbO–Na_2O–B_2O_3$, the enamels just described will lie within a lozenge-shaped field bounded by the following limits: boric oxide, 8 to 11 per cent.; lead oxide, 0 to 6 per cent.; and sodium oxide, 21 to 24 per cent. These enamels are fairly free from the ordinary enamel defects, as compared with the rest of the group or the other groups. They have good resistance to solution as tested with cold 20 per cent. hydrochloric acid and fire down smoothly at 850 to 900° C. (about 1560 to 1650° F.). They are, however, rather difficult to melt and to grind.

NOTES FROM THE RESEARCH LABORATORY, EASTMAN KODAK COMPANY.*

GRAIN GROWTH IN SILVER HALIDE PRECIPITATES.[1]

S. E. Sheppard and R. H. Lambert.

PARTICULARS are given of an experimental study of the grain growth of silver bromide precipitates which shows that the process is regulated by definite quantitative laws. Two principal periods are distinguished, the first consisting in nucleation of the new solid phase; the second, in the growth of grains.

Precipitation was accomplished by mixing silver nitrate and potassium bromide, each containing gelatin, at the juncture of a Y-tube; the mixed material was then ripened at constant temperature by digestion in a solution containing excess of potassium bromide.

Samples were removed from time to time and grain sizes were measured photomicrographically. Following a statistical analysis, examination now indicates that the grains in the first classes (small grains) decrease proportionately to the number present at any given time. A break in the curve, however, occurs at a time independent of excess soluble bromide but depends on the size of grains. Growth before the break is attributed to Ostwald ripening. After the break, the growth is attributed to collision and recrystallization.

A theory of grain growth is suggested involving a double layer about individual grains through hydration of imperfectly oriented ions of silver bromide and of the solubilizing potassium bromide. This explains the effective dispersion of grains during Ostwald ripening and subsequent coalescence, orientation, and recrystallization.

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* Communicated by the Director.

[1] Communication No. 357 from the Kodak Research Laboratories and published in the *Colloid Symposium Monograph*, 6: 265. 1928.

THE FOGGING BY ACIDS AND OXIDIZING AGENTS AND THE INTENSIFICATION OF PHOTOGRAPHIC LATENT IMAGE.[2]

E. P. Wightman.

FURTHER arguments are given in support of the viewpoint of Wightman (*Brit. J. Phot.*, **74**, 447 (1927)) that the cause of latent fog formation from the action of certain oxidizing agents and acids is probably caused by the formation of bromine from the soluble bromide in the plate, with which hypothesis Lüppo-Cramer is in disagreement (*Z. wiss. Phot.*, **25**, 121 (1928); *Phot. Korr.*, **64**, 49 (1928)). The hypothesis was originally based on the fact, discovered by Lüppo-Cramer, that very dilute bromine solution causes fog and on a previously proposed mechanism of latent image intensification and fog formation by hydrogen peroxide, further evidence for which is now presented.

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[2] Communication No. 366 from the Kodak Research Laboratories and published in *J. Amer. Chem. Soc.*, **50**: 2923. 1928.

NOTES FROM U. S. BUREAU OF MINES.*

WORLD'S PRODUCTION OF COPPER INCREASING AT RAPID RATE.

THE world's production of copper during 127 years, since the beginning of the nineteenth century, amounts to more than 40,000,000 tons, according to the United States Bureau of Mines, Department of Commerce. In comparison, all previous production since the beginning of man's utilization of copper—estimated by some as less than a million tons—is relatively insignificant.

The important part played by the United States in supplying the world with copper is interesting and significant, it is pointed out in an economic study of the production of copper during the nineteenth and twentieth centuries, which has just been completed by C. E. Julihn and the Common Metals Division of the Bureau of Mines. The United States has produced more than 19,500,000 tons of copper, or 48 per cent. of the world's output since 1800, although the production of this country was negligible prior to 1850. The increase in rate of United States production from 1845 to 1917 was even greater than the increase in world production during the same period. During the six decades ending with 1920 the average increase per decade in world production was 60.1 per cent. During the same period the average increase in the United States production was 145.8 per cent.

The United States began to produce copper in the decade ending with 1850; the quantity produced was less than 1 per cent. of the world total. Following that decade, however, the increase was rapid up to the end of the nineteenth century, during the last decade of which the United States produced 52 per cent. of the world total. Since then the relation of the United States production to that of the world has remained fairly constant.

Of the world's output of copper since 1800, about 1¼ per cent. was produced in the 25-year period between 1801 and

* Published by permission of Director of the Bureau of Mines.

1825; 2½ per cent. between 1826 and 1850; 5¾ per cent. between 1851 and 1875; 18¾ per cent. between 1876 and 1900; and 63½ per cent. between 1901 and 1925. About 8¼ per cent. was produced during the years 1926 and 1927.

Compared with the first decade of the nineteenth century, world production of copper for the first decade of the twentieth century had increased forty fold, and sixty-seven fold for the succeeding 10 years. For more than 100 years there has been an average increase of 53 per cent. per decade in the rate of production. Only in the period 1919 to 1925 was acceleration of world production interrupted.

The past 127 years have witnessed a geographical shift in the production of copper. During the first half of the nineteenth century Europe produced 63 per cent. of the world's output; South America furnished 17 per cent., Asia 13 per cent., and North America 5 per cent. During the next 50 years the North American output was 37 per cent. of the world total; European production amounted to less than 30 per cent., South American 21 per cent., and the remainder was produced by Australasia, Asia, and Africa. In the first 25 years of the present century North America produced 66 per cent.; South America 11 per cent.; Europe 10 per cent.; and Africa, Asia and Australasia the remainder. Statistics for 1926–27 indicate that North America still retains first place in world copper production with 61 per cent., South America second with 18 per cent., Europe third with 8 per cent., Africa fourth with 7 per cent., and Asia fifth with 5 per cent.

The first important copper production in the United States came from the Lake Superior district in Michigan. This State continued to be the most important source until 1887, when it was surpassed by Montana. In 1907 Arizona became the leading producer and with the exception of one year has held that position to date. Utah became the fourth largest producer of copper in 1903 and in 1926 ranked second.

The results of the Bureau's study are given in Economic Paper No. 1, "Summarized Data of Copper Production." While the Bureau publishes annual reports of the production of the various industrial metals, detailed long-time records of production have not been available. For such studies the facts of production from at least the early years of the nine-

teenth century are desirable, as they serve to link the steadily expanding production of that century with the relatively small production of all previous centuries; but for most of the metals no such record is available. Details of world production of most metals previous to the middle of the last century are decidedly sketchy, and even subsequent to that time the record leaves much to be desired.

The present report on copper is the first of a proposed series of economic papers to be prepared by the Bureau's Common Metals Division, which will present similar analyses of long-time production of other metals such as lead, zinc, iron, chromium, manganese, nickel, aluminum, tin, gold, and silver. Another group of papers on resources is in contemplation, and eventually the Bureau hopes to present papers dealing with distribution of industrial flow and ultimate consumption. Economic papers discussing the opportunities for American citizens or corporations to exploit foreign mineral deposits will also be prepared.

DEVELOPMENT OF AN AMERICAN MINERAL PAINTS INDUSTRY.

A PROGRAM of investigation which promises to assist in the development of a substantial American industry in a field hitherto largely neglected is the study of the ochers and mineral pigments of this country being conducted by the United States Bureau of Mines, Department of Commerce, at its Northwest Experiment Station, Seattle, Washington. This study, which was first concerned with the ochers and mineral pigments of the Pacific Northwest States, is being extended to cover all the known ocher deposits in the country. Coöperation has been obtained from most of the state geological departments, and they are collecting samples and studying the occurrence of the pigments in their respective states. The value of an ocher depends principally on its color and the closer it approaches the golden yellow hue of the French ocher, the greater will be its value. When the original sand and impurities have been removed by washing in water, the ocher should have an impalpable fineness and an iron oxide content of 17 per cent. or more, and should be able to retain a large portion of its color strength when diluted with zinc oxide.

A report on "The Ochers and Mineral Pigments of the Pacific Northwest" is now in press. The report covers the occurrence; methods of purification; and tests of color, oil absorption, and dilution with zinc oxide. Yellow ocher from near Spokane, Washington, and red and brown pigments from other parts of the Northwest were studied. Other colors were produced by heating the ochers and siennas alone or after intimate mixing with manganese ore from the Olympic peninsula of Washington. Some of the ochers studied possess light yellow hues similar to the imported French ocher.

All interested parties are invited to send one-pound samples of ochers or other pigment materials to the Northwest Experiment Station, United States Bureau of Mines, Seattle, Washington, for preliminary classification tests.

EXTRACTION OF POTASH AND OTHER VALUABLE PRODUCTS FROM GREENSANDS.

AN investigation of some of the various processes proposed for the utilization of greensands, of which there are very large deposits along the Atlantic seaboard, is in progress at the Nonmetallic Minerals Station, of the United States Bureau of Mines, at New Brunswick, New Jersey. A bibliography of the literature on greensands has been completed and will be published in the near future. An economic survey of some of the proposed processes is also in course of preparation.

In the laboratory, experiments have been in progress for about two months on the so-called Charlton-Shreve process, involving the digestion of greensands with slaked lime at 200° C. The results of these experiments may be briefly summarized as follows:

Digestion for two hours of 1 part greensand with 0.9 parts of lime, and 5.0 parts of water at 200° C. and 225 pounds pressure in an autoclave resulted in 55 per cent. of the potash being rendered soluble. Digestion for four hours rendered soluble 67 per cent. of the potash. The greensand used contained 6.5 per cent. potash, and was ground as fine as is possible in a small ball-mill. These results agree reasonably well with those of previous investigators.

PATH OF THE STOCK IN BLAST FURNACES.

THE proper preparation each day of some 1,600 to 1,800 tons of iron ore, 500 to 600 tons of limestone, and 600 to 800 tons of coke in the upper 60 feet of a modern blast furnace depends upon contact with about 100,000,000 cubic feet of gas which during 24 hours moves counter-current to the foregoing quantities of descending stock. Equal descent of the stock and uniform gas flow would be ideal for producing maximum gas-solid contact.

For several years the United States Bureau of Mines has been accumulating at its North Central Experiment Station and in the field information on the path of materials in blast furnaces. Gas samples from the tuyeres of 13 furnaces furnished the first definite information that the stock was likely moving faster along the furnace periphery than in the center. Faster movement of stock along the walls seemed a logical result of localized combustion, disclosed by gas samples removed through the tuyeres. Measurements of gas velocities at several levels of an industrial furnace, together with fundamental laboratory data on the flow of gas through beds of broken solids, made it possible to calculate the size distribution of stock across several planes. Such calculations show that the particle size is more uniform across lower levels than at the stock line. As the materials are charged they segregate according to particle size. Mixing of sizes takes place as the stock descends. Particles in the center of the furnace move laterally toward the line of fastest movement, just as a piece of driftwood moves toward the position of a stream where the current is moving most rapidly.

Faster movement of stock along the periphery of the furnace has been further substantiated by stock flow measurements in a furnace. The average rate for 3 points next to the wall was 3 inches per minute, while the average rate near the center was 2.1 inches per minute. Other data have been obtained on conditions inside of the furnace. Such information is needed to point the way toward greater smelting efficiency.

Cod Liver Meal. Cod liver meal is a by-product of the manufacture of cod liver oil, and is used as a feedstuff. R. M. BETHKE, G. ZINGALIAN, D. C. KENNARD, and H. L. SASSAMAN of the Ohio Agricultural Experiment Station (*Jour. Agric. Research*, 1928, *36*, 747–753) have made a series of biological tests of cod liver meals prepared from fresh cod livers after extraction of the oil. They used chickens and rats as experimental animals, and studied particularly the antirachitic value of the meals; this is a measure of the content of fat-soluble D vitamin.

Several samples of the meal showed differences in chemical composition and in antirachitic value. This value was not proportional to the fat content of the meal; and the ether extract or fat obtained from the meal was less potent than ordinary cod liver oil. The conclusion is drawn that cod liver meal should not be used as a substitute for cod liver oil as an antirachitic in the production of poultry and livestock.

J. S. H.

Benzene Poisoning as a Possible Hazard in Chemical Laboratories. J. J. BLOOMFIELD of the U. S. Public Health Service (*Pub. Health Rep.*, 1928, *43*, 1895–1897) suggests that benzene poisoning may occur in laboratories where benzene (benzol) is used as a solvent in the analysis of such materials as rubber, paint, varnish and oil products, and in the cleaning of apparatus, or the removal of stains from the skin. In industrial plants, where the exposure to benzene vapor is continuous, a concentration of benzene as low as 100 parts per million of air is a substantial hazard. However, in laboratories, the use of benzene usually is intermittent. In chronic benzene poisoning, the leucocyte or white cell count of the blood is reduced to a considerable extent, the count falling to values below 5,000 per cubic millimeter. The differential leucocyte count, or the relative numbers of the various types of white cells, also undergoes changes; the per cent. of lymphocytes shows a relative increase, and the per cent. of polymorphonuclear cells a marked decrease. It is recommended that benzene be used only as a solvent, never as a cleansing agent. The further recommendation is made that it be replaced as a solvent by less toxic compounds, such as toluene, xylene, or high-flash naphtha. The health of the laboratory workers should be checked by means of blood counts made at intervals of one or two months.

J. S. H.

THE FRANKLIN INSTITUTE

STATED MONTHLY MEETING, DECEMBER 19, 1928

THE stated monthly meeting of the Institute was called to order at eight-twenty-five by the Secretary, Dr. Howard McClenahan.

The Chairman announced that this was the regular monthly meeting and stated that the minutes of the preceding meeting had been printed in full in the December number of the Journal of the Institute. He requested that these minutes be approved as printed. Upon motion the minutes were so approved.

The Chairman announced that since the last meeting, four Non-Resident members had been elected to the Institute and announced also, with regret, the death of three members.

The Chairman stated that nominations for the election of officers for the ensuing year were in order at this meeting, and called for such nominations. He quoted the By-law of the Institute governing such nominations, and then presented the following names:

Office	Nominee	Term	Nominated by
President	Mr. Nathan Hayward	One year	Mr. Henry Howson Mr. S. T. Wagner Mr. J. S. Rogers Mr. C. A. Hall
Vice-President	Mr. C. C. Tutwiler	Three years	Mr. E. H. Sanborn Mr. W. G. Littleton
Treasurer	Mr. Benjamin Franklin	One year	Dr. J. S. Hepburn Mr. R. W. Lesley
Board of Managers	Dr. James Barnes Mr. G. H. Benzon, Jr. Mr. G. H. Clamer Mr. Theobald F. Clark Dr. A. W. Goodspeed Mr. Charles Penrose Dr. G. D. Rosengarten Mr. W. C. Wetherill	Three years	Dr. G. A. Hoadley Mr. L. F. Levy Mr. S. Shoemaker

The Chairman then presented the resignation of Mr. Nathan Hayward from membership in the Board of Managers and requested that it be accepted. It was then moved and seconded that this action be taken and the motion was unanimously carried.

The Chairman stated that a member of the Board of Managers should be nominated at this time, to serve for the unexpired term of Mr. Hayward, and presented the name of Mr. Malcolm Lloyd of Philadelphia, he was nominated by Mr. Morris L. Clothier and Mr. James L. Whitaker.

The Chairman called for other nominations for any of the offices or for membership in the Board of Managers. No other nominations being offered, the Chairman declared the nominations closed and stated that a ballot would be prepared in accordance with the nominations which had been offered, for submission to the membership of the Institute, in advance of the January monthly meeting.

The Chairman stated that two John Scott Medals and Awards were to be presented at this meeting, to Dr. H. D. Arnold of the Bell Telephone Laboratories, Inc., and to Mr. James C. McBride of the Worthington Steam Pump Company, members of The Franklin Institute, and to this end introduced Mr. Louis Heiland, Secretary of the Board of Directors of City Trusts, to take charge of the meeting during these presentations.

Mr. Heiland called upon Mr. McBride and presented him with the Certificate of Award, the John Scott Medal, and a Premium of one thousand dollars, for the development of the locomotive feed water heater.

He then called upon Dr. H. D. Arnold and presented him with the Certificate of Award, the John Scott Medal, and a Premium of one thousand dollars, for the development of the three electrode high vacuum thermionic tube.

Proper acknowledgments of the presentations were made by the two recipients of them. Both Dr. Arnold and Mr. McBride are members of The Franklin Institute and had requested that the presentations be made before the Institute.

The Chairman, Dr. McClenahan, then resumed the Chair, and presented as the lecturer of the evening Dr. Hermann J. Muller, of the University of Texas, who spoke on "The Enigma of the Gene and of Its Mutations." Dr. Muller spoke delightfully concerning his subject, telling of his own important researches in this field of thought. The lecture was followed by an animated discussion for about a half hour, after which the meeting adjourned with a vote of hearty thanks to Dr. Muller for his interesting talk.

HOWARD MCCLENAHAN,
Secretary.

COMMITTEE ON SCIENCE AND THE ARTS.

(*Abstract of Proceedings of Stated Meeting held Wednesday, December 5, 1928.*)

HALL OF THE INSTITUTE,
PHILADELPHIA, December 12, 1928.

DOCTOR GEORGE S. CRAMPTON *in the Chair.*

The following reports were presented for final action:

No. 2895: Masonite.

The device considered in this report is for the production of disintegrated wood fibers and their utilization in the manufacture of useful materials. The fibers are made by running the waste wood from sawmills through a machine that reduces it to small chips and by inclosing them in a hollow cylinder capable of withstanding high pressure.

Superheated steam is introduced into the cylinder and after a sufficient time for the steam to penetrate and soften the chips a gate is opened in the base of the cylinder and this is of such a shape that when the steam pushes out the chips they are compressed and the pressure being removed suddenly, the absorbed steam explodes the wood into its fibers—the result being a fibrous mass.

This is made into felted sheets from a water bath and the excess water is removed by passing these sheets between rollers or by drying. They are then cut into convenient lengths and placed between the steam heated upper and lower platens of a press.

The resulting product has a smooth surface, the thickness and density of the inner portion depending upon the temperature of the platens, the pressure employed and the length of the treatment.

The material obtained by this process is either "Masonite," an insulation board about a half inch in thickness having dense surfaces and fibrous interior, or "Presdwood," a much thinner board of uniform density and having surfaces that can be finished by standard methods. Both the products are in extensive use for construction and industrial application.

The John Price Wetherill Medal was recommended to Mr. William H. Mason, of Laurel, Mississippi, "in consideration of his process for the successful use of wood waste."

No. 2897: The Work of Mr. W. H. Gartley in the Gas Industry.

This report gives, in detail, a statement of Mr. Gartley's forty-five years devoted to the gas industry, calling attention to his inventions of devices that have been of direct application in the industry, to his work as chief engineer of the Philadelphia Gas Works and to his papers presented before the American Gas Institute.

He was awarded the Beal Medal by this Institute for his paper on the "Delivery of Uniform Candle Power to the Consumer," this award being for "the best paper at the American Gas Institute Meeting of 1906."

During the World War he was a member of the Committee on Coal Tar By-Products for the Chemical Committee of the Council of National Defense. His expert knowledge of the manufacture of toluol was of great service to the War Industries Board as was also the production of toluol by the Gas Works of Philadelphia under his engineering operation.

The award of the Walton Clark Medal was recommended to Mr. W. H. Gartley "In consideration of his engineering contributions to the advancement of the gas industry, to the development and practical operation of large city gas works, of his ingenuity, sound judgment, and service to the public in his field of work."

The following report was presented for first reading:

No. 2879: Temperature Control in Gas Furnaces.

GEO. A. HOADLEY,
Secretary to Committee.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting, Board of Managers, December 12, 1928.)

NON-RESIDENT.

MR. HOWARD W. AMBRUSTER, Consultant and Importer, 261 Broadway, New York City. For mailing: 597 Highland Avenue, Westfield, N. J.

MR. CARL T. HOLMQUIST, Vacuum Tube Engineer, 1850 Otto Street, Chicago, Illinois. For mailing: 529 North Leclaire Street, Chicago, Illinois.

MR. MALCOLM M. HUBBARD, Student—Massachusetts Institute Technology, Cambridge, Mass. For mailing: 8 Cottage Street, Newport, R. I.

DR. J. C. MCLENNAN, Director of Physical Laboratory, Professor of Physics University of Toronto, Toronto, Canada.

CONTRIBUTING.

Representing American Engineering Corporation.

MR. H. WELLS, Llanerch, Pa.

Representing Bell Telephone Company.

MR. D. S. HILBORN, Engineering Department, Bell Telephone Company, 1835 Arch Street, Philadelphia, Pa.

CHANGES OF ADDRESS.

MR. MILTON W. DEISLEY, 1829 West Venango Street, Philadelphia, Pa.

MR. J. E. EGLESON, Old Chester Road, Essex Fells, N. J.

MR. CHARLES HART, Wrought Iron Company of America, 732 Commercial Trust Building, Philadelphia, Pa.

MR. ERNEST L. HOPPING, S. W. corner Ninth and Sansom Streets, Philadelphia, Pa.

DR. CHARLES H. HERTY, 101 Park Avenue, Room 1834, New York City.

MR. ROBERT JOB, 980 St. Antoine Street, Montreal, Quebec, Canada.

MR. MICHAEL MELODY, 506 East Durham Street, Mt. Airy, Philadelphia, Pa.

MR. JAMES WARNOCK, JR., 511 Chestnut Street, Philadelphia, Pa.

NECROLOGY.

Mr. Strickland L. Kneass, Philadelphia.
Mr. T. P. Chandler, Radnor, Pa.
Mr. J. Snowden Bell, New York City.

LIBRARY NOTES.

RECENT ADDITIONS.

American Mathematical Society. Bulletin. Volumes 1–33. Thirty-three volumes. 1895–1927.

American Mathematical Society. Transactions. Volumes 1–29. Twenty-nine volumes. 1900–1927.

ARMITAGE, F. P. A History of Chemistry. 1912.
ARRHENIUS, SVANTE. Theories of Chemistry. 1907.
BRAGG, SIR WILLIAM H. The Structure of an Organic Crystal. Fison Memorial Lecture, 1928.
BROGLIE, LOUIS DE, AND LEON BRILLOUIN. Selected Papers on Wave Mechanics. 1928.
CHATFIELD, CHARLES HUGH, AND CHARLES FAYETTE TAYLOR. The Airplane and Its Engine. First edition. 1928.
COHEN, LOUIS. Heaviside's Electrical Circuit Theory. 1928.
Colloid Symposium Monograph. Volume 6. 1928.
CROFT, TERRELL. Practical Heat. First edition. 1923.
DOLBEAR, A. E. Modes of Motion: or Mechanical Conceptions of Physical Phenomena. 1897.
EDMUNDS, E. W., AND J. B. HOBLYN. The Story of the Five Elements. 1911.
GEIGER, H., AND K. SCHEEL, editors. Handbuch der Physik. Band 6. 1928.
GIBSON, CHARLES R. Modern Conceptions of Electricity. 1928.
HACKH, INGO W. D. Chemical Reactions and Their Equations. Second edition, revised. 1928.
Handbook Encyclopedia of Engineering. First edition. 1928.
HOOVER, HERBERT C. Principles of Mining. First edition. 1909.
HOPKINS, NEVIL MONROE. The Outlook for Research and Invention. 1919.
INGALLS, ALBERT G., editor. Amateur Telescope Making. 1928.
LIBBY, WALTER. An Introduction to the History of Science. 1917.
LIDDELL, DONALD M., editor. Handbook of Non-Ferrous Metallurgy. Prepared by a staff of specialists. Two volumes. First edition. 1926.
LINDGREN, WALDEMAR. Mineral Deposits. Third edition. 1928.
LODGE, SIR OLIVER. Ether and Reality. 1925.
MOYER, JAMES A., AND RAYMOND U. FITTZ. Refrigeration. Including Household Automatic Refrigerating Machines. First edition. 1928.
National Research Council. Annual Survey of American Chemistry. Volume 3, July 1, 1927 to July 1, 1928.
POINCARÉ, LUCIEN. The New Physics and Its Evolution. 1908.
PORTER, C. W. Molecular Rearrangements. 1928.
PURDAY, H. F. P. Diesel Engine Design. Third edition. 1928.
RICHTMYER, F. K. Introduction to Modern Physics. 1928.
Scientific American Cyclopedia of Formulas. Edited by Albert A. Hopkins. 1928.
SEYMOUR, HENRY. The Reproduction of Sound. No date.
SOTHERN, J. W. M. Marine Diesel Oil Engines. Third edition. 1928.
STUDLEY, BARRETT. Practical Flight Training. 1928.
SUMNER, P. H. The Science of Flight. Two volumes. 1926–1929.
THOMSON, SIR J. J. Beyond the Electron. 1928.
WHYTE, CHARLES. Our Solar System and the Stellar Universe. 1923.
Wien-Harms Handbuch der Experimentalphysik. Volume 7, Pt. 1; Volume 13, Pt. 2; Volume 15; Volume 18; Volume 23, Pt. 2. Five volumes. 1928.
WILES, ROBERT. Cuban Cane Sugar. 1916.

BOOK REVIEWS.

MOLECULAR REARRANGEMENTS. By C. W. Porter, Professor of Chemistry in the University of California. 167 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1928. Price, $4.

Research is steadily penetrating into the intimate structure of matter. The ultra-microscope with its interesting revelation of colloidal phenomena is now very familiar. In another direction we have the study of atomic arrangements by means of x-rays, these investigations having brought about many changes in our views as to the molecular structure of compounds. A still more intimate study is as to the transformations that occur within a given molecule by rearrangement of its atoms. Often such investigations proceed irregularly and encounter scornful treatment by those who should consider the matter carefully and impartially. The history of science shows a good many such arbitrary condemnations. When Newlands presented to the Chemical Society his suggestions as to the significant relations of the atomic weights of certain groups of elements he was criticised severely, and his communication was refused publication in the Society's Journal. Yet to-day the arrangement of the elements according to atomic weights is one of the most prominent features in chemistry. In the preface of this book we are told that certain suggestions in regard to internal rearrangements were derisively disapproved as the "flip-flop theory."

Wöhler's experiment showing the conversion of ammonium cyanate into urea was the first observed case of molecular rearrangement but it was more particularly famous as the first production from inorganic sources of a compound then known only as derived from living organisms. The text of this work shows a very large number of transformations of this type, but of course many far more complicated than Wöhler's. The phenomena are discussed in seven chapters and all the changes are amply illustrated by structural formulas. The work constitutes therefore, a very comprehensive presentation of inter-molecular transformations. There are, of course, border-lands which may be disputed areas, but the author has been careful to keep within certain specific lines and has presented a very interesting and valuable summary of some of the special developments of organic chemistry.

HENRY LEFFMANN.

AMATEUR TELESCOPE MAKING. Albert G. Ingalls, Editor, with contributions by Russell W. Porter, Prof. Charles S. Hastings, Ph.D., Rev. William F. A. Ellison, F.R.A.S., Dr. George Ellery Hale, Sc.D., Dr. Elihu Thomson, Ph.D., Clarendon Ions, John M. Pierce, and A. W. Everest. With a foreword by Dr. Harlow Shapley. 285 pages, illustrations, 8vo. New York, Scientific American Publishing Company, 1928. Price, $3.

The productions of amateurs in a diversity of fields are often of a quality that might easily earn their originators recognition in professional ranks, and the value of amateur work, not alone as a diversion of absorbing interest but as a real aid in the choice of a professional career has long been widely appreciated. The Scientific American which may rightfully be termed the dean of popular scientific journalism in this country, for several generations has been at the forefront in fostering the interest of amateur devotees of science as well as providing a valuable

fund of information on progress in every branch of science. One highly interesting activity of this enterprising journal in recent years is the issue of a number of articles which deal with the construction of astronomical reflecting telescopes of small size adapted in design to the requirements and capacity in craftsmanship of amateurs. In harmony with the interest aroused in the subject by these articles, the Editor was induced several years ago to seek the coöperation of the noted authors whose names appear on the title-page of this volume, in the contribution of articles covering every phase of the construction of reflecting telescope for compilation into book form. The present treatise, now in its second edition is the result.

Consisting of separately written sections, the book necessarily lacks the continuity of a formal text. There is also a certain amount of repetition which, however, is helpful rather than detrimental in emphasizing critical processes by discussion from more than one point of view. Part I by Russell W. Porter and Part II by the Rev. William F. A. Ellison are veritable condensed treatises on the subject. Detailed directions are given for the process of grinding the concave mirror by means of the pitch lap and the foucault-knife-edge test for determining its accuracy. Equally minute directions are to be found for the production of prisms, flats and making the lenses of eye-pieces as well as the mounting for these elements. Methods of silvering glass receive the full treatment which its importance deserves. The material in Part II is reprinted from Doctor Ellison's The Amateur Telescope, an English publication. Among the special contributions of the different authors may be mentioned: silvering, teloscopic oculars, grinding and polishing machines, special designs of telescope, solar phenomena and the spectroheliоscope and an extensive collection of miscellaneous data. The book as a whole is a unique collection of useable information not available elsewhere in a single work.

LUCIEN E. PICOLET.

ANNUAL SURVEY OF AMERICAN CHEMISTRY, Volume 3. Prepared under the auspices of the Division of Chemistry and Chemical Technology National Research Council. Edited by Clarence J. West, Director, Research Information Service National Research Council. 395 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1928. Price, $3.

The good work still goes on actively. The development of American chemical research in the years immediately following the war was an encouraging feature, but there was a fear that as the necessity for such research became less as far as regards the general public, there would be a cessation of activity. This seems so far not to have occurred. Three volumes embodying the principal results of American investigation have now appeared, and it would seem from the present treatise that there is no abatement in the zeal in which the work is being carried on. Any one who knows the value of systematic research will be gratified by this fact. The arrangement of the present volume is similar to that of the preceding ones. The editorial board has adopted the policy of varying somewhat the subjects included, by which certain less active fields will be only occasionally covered. The authors of the essays also will be different in different years, thus giving greater variety to the method of treatment and to the topics presented. When the first volume was issued there were some unfavorable comments, probably mostly by

those who are strongly impressed with the belief that science should be international, and that a summary limited strictly to one nation is inadvisable. The support with which the first two volumes have been met indicates that the system is much favored. It is to be hoped that the volumes will continue, as they will serve not only for the comprehensive report of what is being done in chemistry in its many departments in this country, but as a stimulation to further extension of such research.

The volume is excellently printed and it contains a very large amount of information.

HENRY LEFFMANN.

INTRODUCTION TO MODERN PHYSICS. By F. K. Richtmyer, Professor of Physics at Cornell University. First Edition XV–596 pages, 6 plates, 169 figures, 8vo. cloth. New York, McGraw-Hill Book Company, Inc., 1928. Price, $5.

In this Introduction to Modern Physics the author strives to acquaint us with the prevailing theories in much the same manner as we employ in becoming acquainted with our best friends. Meeting a theory or a person for the first time, we are filled with respect and admiration for their ability and accomplishments. On the other hand, a feeling of intimacy and deep-seated interest towards such entities comes only from a knowledge of their background. Through a study of their past we can follow the slow but steady development from almost nothing at all and the overcoming of this and that difficulty thus acquiring a deeper appreciation of the person or a more complete recognition of the significance of some theory.

The author announces that the book was written for a pedagogica purpose. As such he has employed true pedagogic methods in presenting the subject matter to its readers. Here and there a personal touch may be found taking the form of an epigrammatic expression or an italicized word furnishing a glimpse of the writer's dry humor. All these serve to whet the interest of the student and promote a friendly and receptive attitude toward such phases of scientific knowledge.

In this particular case the term "modern physics" includes those discoveries and theories which have come into being since 1890. In the earlier part of this present period, progress in the delevopment of physics was punctuated by the photoelectric effect, the x-rays, the electron, radio-activity and evolution of the quantum theory. The author quotes: "Indeed, the reconciliation of these two theories, (electromagnetic and quantum) each based on incontrovertible experimental evidence, may be said to be one of the two great problems of modern physics; the other problem being that of the structure of matter."

In keeping with plan of presentation the first three chapters respectively are: Historical Sketches of the First Period: Earliest Times to 1550 A.D.; Second Period (1550–1800 A.D.); The Rise of the Experimental Method and Third Period (1800–1890 A.D.); The Rise of Classical Physics. The chapter on the electromagnetic theory of light should be of particular interest since the author presents in a logical manner the development of the corresponding theoretical equations from the elementary laws of mechanics and integral calculus The first element of doubt concerning the universal applicability of the electromagnetic theory of light is presented in the chapter on the photoelectric effect. A chapter

on the origin of the quantum theory follows which is outstanding for its clear, direct and logical treatment of the subject. With the advent of the quantum theory the way is now open for an interpretation of various observed phenomena in the light of this theory. Among those considered are specific heats, series relations in line spectra, the origin of spectral lines, the nuclear atom, arrangement of electrons in atoms and X-rays. The last chapter is given over to consideration of the second great problem of modern physics, i.e. the structure of matter. Appendix I supplies complete and valuable information concerning the elements, giving the atomic numbers, atomic weights as well as number and weight of the isotopes. Appendix II gives the arrangement of electrons in orbits according to Foote while III contains a list of some important constants.

Of any book in physics written for the purpose of interesting the student in the subject this one should accomplish its purpose with ease. Fortunately Dr. Richtmyer did not turn a deaf ear toward his importune former students and other friends.

T. K. CLEVELAND.

CHEMICAL REACTIONS AND THEIR EQUATIONS. A guide for students of chemistry. By Igno W. D. Hackh, Professor of chemistry, College of Physicians and Surgeons, San Francisco. Second edition, revised. 145 pages, illustrations. 8vo. Philadelphia, P. Blakiston's Sons & Company, 1928. Price, $2.

All who have had experience in teaching chemistry will heartily agree with the statement in the preface of the first edition to this book that students are particularly deficient in ability to balance equations, even simple equations are often turned in unbalanced. A further difficulty is a lack of care in calculations and a general unwillingness to check up work or to examine the calculation carefully to see whether the equation is rational. One is not surprised that the author of this book has found even graduate students at a great university unable to work out a moderately complicated equation. Students of the first and second year are generally very poor in this field. The present work is a summary of the principles of chemistry especially directed to the treatment of chemical reactions. There is no broad discussion of the great principles of chemistry or physical chemistry but there is a very large amount of illustrating material which will serve very well to help students in what is found to be by experience a difficult part of the study of chemistry.

HENRY LEFFMANN.

SOLUBLE SILICATES IN INDUSTRY. By James G. Vail. American Chemical Society Monograph Series. 443 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1928. Price, $9.50.

This is an American Chemical Society Monograph issued by the Chemical Catalog Company, constituting No. 46 of that series, which has already covered in a most excellent manner a wide field of applied chemistry and brought much credit to American science. A Philadelphia reviewer may be pardoned for reacting pleasantly to the fact that the author is of that city, for recent publications in the field of American chemistry have shown few instances in which it has appeared in conspicuous position in either the pure or applied department of the science.

Silicon is one of the most abundant elements of the earth's crust and its oxide

is presented to us in a great variety of forms from the brilliant quartz crystals, and amorphous forms so prized by collectors, to common sand. In one state, the typical crystalloid, the substance that gave the word to our language,—in the other state a typical colloid it finds many applications in pure and applied chemistry.

The volume in hand relates only to the commercial silicates, of which, of course, sodium silicate is the most prominent, and it is still further limited to the soluble salts, so that many products such as glass and porcelain are not mentioned. The author's long and active association with a company manufacturing soluble silicates has given him extensive and accurate knowledge of the manufacturing processes and the many uses of the products. The book is liberally provided with illustrations of apparatus, graphs, and tables. Turning over its pages one is much impressed with the high development to which the manufacture of these silicates has been brought and also the very large number of applications that they have obtained. The comprehensiveness of the work is shown by the fact that space is devoted to considerations of the relations of the compounds to the public health, including a brief note of a case of non-fatal poisoning by sodium silicate.

Typography, press-work and general make-up are in accordnace with the excellent standard of the publishing company and the text is clearly and concisely written.

HENRY LEFFMANN.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 292. Characteristics of Five Propellers in Flight, by J. W. Crowley, Jr., and R. E. Mixson. 23 pages, illustrations, quarto. Washington, Government Printing Office, 1928. Price, fifteen cents.

This investigation was made by the Committee at Langley Field for the purpose of determining the characteristics of five full-scale propellers in flight. The equipment consisted of five propellers in conjunction with a VE-7 airplane and a Wright E-2 engine. The propellers were of the same diameter and aspect ratio. Four of them differed uniformly in thickness and pitch and the fifth propeller was identical with one of the other four with the exception of a change of the airfoil section. The propeller efficiencies measured in flight are found to be consistently lower than those obtained in model tests. It is probable that this is mainly a result of the higher tip speeds used in the full-scale tests. The results show also that because of differences in propeller deflections it is difficult to obtain accurate comparisons of propeller characteristics. From this it is concluded that for accurate comparisons it is necessary to know the propeller pitch angles under actual operating conditions.

R.

PUBLICATIONS RECEIVED.

Industrial Chemistry, An Introduction. An elementary treatise for the student and general reader, by Emil Raymond Riegel, Ph.D. 649 pages, illustrations, portrait, 8vo. New York, The Chemical Catalog Company, Inc., 1928, price $9.00.

A Survey of Nitrocellulose Lacquer, by Bruce K. Brown and Francis M. Crawford. 368 pages, 8vo. New York, The Chemical Catalog Company, Inc., 1928, price $7.50.

Outlines of Paint Technology, by Noël Heaton. 400 pages, illustrations, table, 8vo. London, Charles Griffin & Company, Limited; Philadelphia, J. B. Lippincott Company, 1928, price $8.00.

Enamels: Their Manufacture and Application to Iron and Steel Ware, by Laurence R. Mernagh. 234 pages, illustrations, plates, diagrams, tables, 8vo. London, Charles Griffin & Company, Limited; Philadelphia, J. B. Lippincott Company, 1928, price $7.00.

Amateur Telescope Making, by Albert G. Ingalls, Editor. 285 pages, illustrations, 12mo. New York, Scientific American Publishing Co., 1928, price $3.00.

Colloid Symposium Monograph Papers presented at the Sixth Symposium on Colloid Chemistry, University of Toronto, June 1928, edited by Harry Boyer Weiser. 346 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1928, price $6.50.

Molecular Rearrangements, by C. W. Porter. American Chemical Society Monograph Series. 167 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1928, price $4.00.

Electrical Engineering Laboratory Practice, by Oskar E. Edison, E.E., and Ferris W. Norris, E.E. 276 pages, illustrations, 8vo. Boston, Ginn and Company, 1928, price $2.80.

Annual Survey of American Chemistry, Vol. III, July 1, 1927, to July 1, 1928, prepared under the auspices of the Division of Chemistry and Chemical Technology, National Research Council. Edited by Clarence J. West, Director Research Information Service, National Research Council. 395 pages, 12mo. New York, The Chemical Catalog Company, 1928, price $3.00.

La Physique de la Gravitation et la Dynamique de L'Univers, par Thomas Tommasina. 300 pages, potrait, quarto. Paris, Gauthier-Villars et Cie., 1928.

Soluble Silicates in Industry, by James G. Vail. American Chemical Society Monograph Series. 443 pages, illustrations, 8vo. New York, The Chemical Catalog Company, Inc., 1928, price $9.50.

Lichtelektrische Erscheinungen, von Bernhard Gudden. 325 pages, illustrations, 8vo. Berlin, Julius Springer, 1928, price in paper, 24 marks.

National Advisory Committee for Aeronautics. Technical Notes, No. 296, Bearing Strength of Wood under Steel Aircraft Bolts and Washers and Other Factors Influencing Fitting Design, by G. W. Trayer. 25 pages, tables, diagrams, quarto. No. 299, The Effect of Fillets between Wings and Fuselage on the Drag and Propulsive Efficiency of an Airplane, by Melvin N. Gough. 4 pages, plate, diagrams, quarto. No. 300, The Variation in Pressures in the Cockpit of an Airplane in Flight, by Thomas Carroll and William H. McAvoy. 6 pages, illustrations, quarto. No. 301, Drag and Cooling with Various Forms of Cowling for a "Whirlwind" Engine in a Cabin Fuselage, by Fred E. Weick. 25 pages, plates, quarto. Washington, Committee, 1928.

CURRENT TOPICS.

Airplane Sign Posts. The Journal has been asked to coöperate with The Daniel Guggenheim Fund for the Promotion of Aëronautics in a nation-wide campaign for roof-markings which will identify the country's towns to the aviator. Such identification is a first requisite for an air transportation system, which will heighten the efficiency of American business by swifter means of communication.

In asking that industrial corporations should assist in roof-marking their buildings, where practical, Harry F. Guggenheim states: "Sign posts of this kind for the airplane are an absolutely essential item in the safety of air transportation. The need for them has been repeatedly stressed to the Fund by Colonel Charles A. Lindbergh as a result of his experience during his United States tour a year ago, and in subsequent cross-country flying. In the opinion of Colonel Lindbergh, who is a technical adviser to the Fund, this identification represents one of the most worth while steps that can be taken for the advancement of civil aviation. His opinion is shared by other pilots from coast to coast and has the emphatic endorsement of all those interested in aëronautics."

As a basis for the work, the Fund, supported by the Postmaster General, has asked some 8,000 postmasters in towns with a population between 1,000 and 50,000 to take the initiative in seeing that their communities are identified by roof-markings. The postmaster is urged to enlist the support of civic organizations such as the American Legion, the service clubs, or the Chamber of Commerce, or individual corporations. To the organization or person responsible for the identification the Fund has asked Colonel Lindbergh to send a certificate of appreciation. The record of roof-markings will be kept by the Fund and made available to the Department of Commerce, the Post Office Department and other government agencies. The movement has the backing of the Departments of War, Navy and Commerce.

Many corporations throughout the country, a number of them in this industry, have already been active in marking the roofs of their own buildings for purposes of aërial navigation. This is not only a civic act helpful to the community in which the corporation is located; it is also of direct benefit to business as a whole, since it is the latter which will benefit particularly from a commercial air transportation system. The Fund asks that a corporation with an available roof in an unmarked town of between 1,000 and 50,000 population should communicate directly with the postmaster.

In a printed bulletin the Fund describes the type of roof most suitable for marking and urges the use of block letters in chrome yellow with a black background. It is suggested that it should be from 10 to 20 feet in height and on roofs preferably of tiled shingles, tin or other metal, or slate where the visibility is not reduced by smoke. Besides the name of the town or city, the marking should include an arrow pointing due north with the letter "N" over it and a smaller letter indicating the airport if there is one.

Effect of Magnetic Fields upon Thermal Conductivity of Iron, Gold, Copper, Silver and Zinc. (*Physical Review*, Sept., 1928.) It is for the ferro-magnetic metals, iron and nickel, that the effect of magnetic fields has been chiefly studied, though much work has been done on bismuth and tellurium. For these the application of a field decreases the thermal conductivity.

A "bar method" was used, the thermal conductivity being given by the temperatures of two points on the bar. The bars of metal, slightly longer than 10 cm. and of cross-sectional area ranging from .051 to .086 sq. cm., were heated by an electric current. This was alternating for all except iron for which a direct current was used on account of the skin effect. Both ends of the bars as well as the walls of the vessel surrounding the bars were maintained at the temperature of melting ice. Thermo-couples, spot-welded to the metal of the bars, gave the temperatures. The current was adjusted so that a parabolic distribution of temperature was attained. The electro-magnet had a 10.7 cm. gap and, when supplemented by a solenoid about the gap, produced a field of 10,000 gauss. The solenoid carried 800 amperes and was oil-cooled. This was for the longitudinal field and could be used with no time limit. The transverse field of 8,000 gauss was obtained by using different pole tips without the solenoid.

Not only the thermal but also the electrical conductivity was measured. The application of a transverse field of 8,000 gauss causes no change in either the thermal or the electrical conductivity of iron, copper, silver, gold or zinc. A transverse field half as strong reduces the thermal conductivity of iron by .4%. No effect upon either conductivity due to the longitudinal field of 10,000 was found for silver, gold and zinc. This field caused an increase in the electrical conductivity of iron and copper amounting to .2 per cent. and .04 per cent. respectively. Both of these metals by the same field suffered losses of thermal conductivity amounting to 1.14 and .23 per cent. respectively. A single crystal of zinc showed no behavior different from ordinary zinc.

G. F. S.

The Physics of the Universe. SIR JAMES JEANS. (*Nature*, Supplement, Nov. 3, 1928.) This was the first Henry Herbert Wills lecture and was delivered at the University of Bristol on Oct. 30, 1928. Quite apart from its scientific value it is an unusual production by reason of its felicitous phrases and pungent illustrations.

Even after geologists came to study the earth as a changing structure and astronomers learned to speculate on the evolution of the stars man still continued to believe the atoms to be immutable. This view persisted till almost the close of the nineteenth century. "Like a child's box of wooden bricks, the atoms made many buildings in turn. Then Crookes, Lenard, and, above all, Sir. J J. Thomson, began to break up the atom." In 1895 Sir J. J. Thomson showed that the fragments chipped off from no matter what atoms, the electrons, are identical, possessing the same mass and equal negative electrical charge. Two years later Lorentz, in explaining the Zeeman effect, made it clear that the moving parts in atomic structures are the same electrons. To Rutherford is due the credit of furnishing a model of the atom that has served for decades to coordinate the facts. A central nucleus with a positive charge is surrounded by negatively charged electrons moving in distant orbits. "The volume of space kept clear by the electrons is enormously greater than the total volume of the electrons; roughly, the ratio of the volumes is that of the battlefield to the bullets. The atom, with a radius of about 2×10^{-8} cm., has about 100,000 times the dimensions, and so about 10^{15} times the volume, of a single electron, of which the radius is about 2×10^{-13} cm. In all probability the 'nucleus' is even smaller than the electrons. The number of orbital electrons in an atom is called the 'atomic number' of the atom." Then the investigations of radioactivity initiated by Becquerel led to the conclusion that the nucleus itself was not an unchanging thing. Given adequate time the nucleus of a uranium atom spontaneously changes into the nucleus of a lead atom and eight alpha particles each of which is the nucleus of a helium atom. With the exceptions of potassium and rubidium radioactivity resides only in the heavy elements, those having atomic numbers larger than 83. Though the lighter elements do not spontaneously disintegrate their atoms can be broken up as Rutherford showed in 1920 by bombarding oxygen and nitrogen with rapid alpha particles. In consequence a vigorous hypothesis exists to the effect that "the whole universe is built up of only two kinds of ultimate bricks, namely, electrons and protons. Each proton carries a positive charge which is exactly equal in amount to the negative charge carried by an electron. The protons are supposed to be identical with the nucleus of the hydrogen atom; all other nuclei are supposed to consist of closely packed

structures of protons and electrons." The atom, however, contains, besides electrons and protons, a third ingredient, electromagnetic energy. "All radiation carries mass about with it, one gram of mass being associated with 9×10^{20} ergs or 2.15×10^{13} calories of radiation. As a necessary consequence, any substance which is emitting radiation must also be losing mass. The ultimate fate of a gram of uranium may be expressed by the equation.

$$1 \text{ gram uranium} = \begin{cases} 0.8653 \text{ gm. lead.} \\ 0.1345 \text{ gm. helium.} \\ 0.0002 \text{ gm. radiation.''} \end{cases}$$

The resulting lead and helium contain just the same protons and electrons as the original uranium. On the earth at least no transformation of radiation into mass is ever found by way of compensation for the change of mass into radiation. The development of heat in the earth due to radioactive processes causes our planet to radiate at a rate that means a loss of mass from it equal to about an ounce a minute.

The Origin of Terrestrial Radium and Uranium.—The average life of radioactive atoms varies from billions of years down to thousandths of a sec. "If a whole earth were built of pure radium only a single atom would be left after a quarter of a million years. Since the earth is many millions of years old, we may be confident that every atom of radium now on earth was born on earth." Radium traces its ancestry back to uranium, which disintegrates slowly. It takes 5,000,000,000 years for this element to diminish to half its original mass. Less than half of this number of years ago the earth was formed from the sun so that most of the earth's stock of uranium at the beginning is now in existence as uranium on the earth. An initial store of about 10^{19} gm. would suffice."

The Ages of the Sun and Stars.—Maxwell showed how to investigate by theory the behavior of a gas whose molecules repelled one another with a force inversely as the fifth power of the distance. If molecules of different masses were present their velocities tended to assume values for which the kinetic energy was the same for all. This adjustment was due to the continuous action of their mutual repulsions. A similar mathematical treatment has been applied to the universe of stars which attract one another with a force inversely as the square of the distance. Here too a time will come when the kinetic energies of the stars will be the same no matter what the mass be and this equipartition of energy will be due to the action of gravitation. However millions of millions of years must elapse before such a time can arrive. A comparison of the kinetic

energies of the stars furnishes ground for the belief that they have about the same value and that in consequence the millions of millions of years have passed already. Such must then be the order of magnitude of the age of the stars. Other considerations based on binary stars lead to about the same age. "The general age of the stars is probably between five and ten million million years."

The Origin of Solar Uranium.—If the entire mass of the sun had been composed of uranium the process of spontaneous disintegration would have reduced the quantity of this element to a single atom within a million million years. If we substitute thorium, the most longlived of the radioactive elements, the corresponding time is three times as great. "When the earth was born the sun's age was greater than either of these times, so that the earth's portion of radioactive matter must have been generated during the sun's life in the sun itself." The author now reaches the question of the manner in which this kind of matter was generated in the sun. Two processes occurring on the earth are considered, viz., change of atomic make-up by rearrangement of electrons and change of nuclear make-up by spontaneous disintegration. They seem at first very different. Nothing helps or hinders the disintegration. Conditions of temperature and pressure make no difference. "On the other hand, the change produced in ordinary atoms by electronic rearrangement is extremely susceptible to external physical conditions. Every spectroscopist knows how to chip off one, two, or even three electrons from the atom at will. Nevertheless, as was first made clear in a remarkable paper which Einstein published in 1917, the difference is merely one of degree and not of kind." An electron can revolve about the central nucleus in any one of a series of orbits of greater and greater dimensions. To cause it to remove from the innermost orbit where its energy is least to the next orbit a definite impartation of energy is requisite. "If it is left entirely undisturbed the electron must, after a certain time, lapse back spontaneously to its original smaller orbit. If it were not so, Planck's well-established law of black-body radiation could not be true. Thus a collection of hydrogen atoms in which the electrons describe orbits larger than the smallest possible is similar to a collection of uranium atoms in that the atoms spontaneously lapse back to their states of lower energy as a result merely of the passage of time, losing mass and emitting radiation in the process." To cause an electron to move from one orbit to another located at a greater distance from the nucleus it is not sufficient to flood the atom with radiation of miscellaneous wave-lengths. On the contrary the atom resembles a penny-in-the-slot machine. It will not work unless the incentive is presented in the right form. A definite wave-length of radiation

causes the electron to move to another orbit and, when it resumes its former orbit, radiation of the same wave-length is emitted. Both the absorption and the emission takes place by quanta which are equal to the frequency of the radiation multiplied by Planck's constant.

The author deduces an interesting geometrical relation "that any structure will only be disturbed by radiation whose wave-length is less than 860 times the dimensions of the structure." Since the nucleus, like the entire atom, is composed of positive and negative charges it ought to behave like an atom toward radiation incident upon it, but the radiation either absorbed or emitted by the nucleus is of much shorter wave-length than that absorbed or emitted by the atom, the atom being 100,000 as large as the nucleus. The quanta effective in disintegrating the nucleus are accordingly by theory 100,000 times as great as those causing rearrangements of electrons. "If we compare the hydrogen atom to a penny-in-the-slot machine, nothing less than five-hundred-pound notes will work the radioactive nuclei."

In passing the author pays his respects to modern alchemists. "In its widest form, their ambition is to combine the electrons and protons of base metals with the third atomic ingredient, namely, electromagnetic energy, so as to form atoms of gold. It would need at least an appreciable fraction of an ounce of energy to produce an ounce of gold, and with electric power at even a farthing per Board of Trade Unit, energy and radiation cost eleven million pounds per ounce. Whatever the gold standard may have to fear on the political side, it would appear to be thoroughly impregnable on the side of physics and chemistry."

It is possible to calculate the temperatures at which bodies will emit predominatingly the wave-lengths of radiation that should be effective in changing atomic and nuclear configurations. "Atoms, for example, whose general dimensions are of the order of 10^{-8} cm., begin to be broken up when the temperature approaches 30,000 degrees; nuclei, whose general dimensions are of the order of 10^{-13} cm., must remain unaffected until the temperature approaches 3,000,000,000 degrees. The shortest wave-length of radiation emitted in the transformation of uranium is about 0.5×10^{-10} cm., and this corresponds to a temperature of 5,800,000,000 degrees." Nowhere in the universe is this temperature attained except possibly in the centers of 'white-dwarf' stars and of spiral nebulæ. Since the sun is far below this temperature it seems impossible that its uranium could have been fabricated from lighter elements. It must have resulted from the disintegration of unknown heavier elements which at one time existed in the sun, and indeed, may still exist there.

"Every square centimeter of the earth's surface discharges radiation out into space at the rate of about 1,500 calories a second, from which we can calculate that the sun's total mass is diminishing at about 250,000,000 tons a minute. Whereas the flow of mass from the earth's surface, a total loss of about an ounce a minute, is about equal to the flow of water from a dripping tap, the flow of mass from the sun's surface is about 150 times the flow of water over Niagara. Many stars lose mass even more rapidly. S. Doradus loses mass at the rate of about 45,000,000 Niagaras." The earth more than recoups its loss by the influx of meteors, but the sun has continued to lose through its long life of 7×10^{12} years, so that it now possesses only a few dozen of each thousand atoms that it originally had. "This transformation of atoms into radiation, although unknown to terrestrial physics, must clearly be one of the fundamental physical processes of the universe. Neither total energy nor total mass is any longer constant; the conservation both of mass and of energy has disappeared from physics, and only a kind of sum of the two is conserved."

The fate of the universe is an interesting question, and on it thermodynamics sheds some light. Two competing schemes of enlarged thermodynamics in general are not in agreement but fortunately they offer concordant results for one limiting case which is that of space almost free from matter. This happens to be the average condition of the universe. "Hubble has estimated that if all the matter within about 100 million light-years of the sun were uniformly spread out, it would have a mean density of the order of only about 10^{-31} gm. per cubic centimeter, so that even the very 'empty' atoms would be at several thousand million times their diameters apart." "Maximum entropy would be attained when every atom has turned into radiation that is capable of doing so. Furthermore there is no place where radiation congeals into matter." There can be no creation of matter out of radiation, and no reconstruction of radioactive atoms which have once broken up. The fabric of the universe weathers, crumbles, and dissolves with age, and no restoration or reconstruction is possible. It will be noted that this conclusion is quite at variance with one recently announced by Millikan.

A few decades ago temperatures of 10,000° seemed rather high but in this paper the radiation accompanying the annihilation or creation of the proton and associated electrons is stated to be characteristic of the temperature of 22×10^{11} degrees.

Elsewhere the author speaks seriously of a temperature of 2×10^{62} degrees.

G. F. S.

Two addresses by Vladimer Karapetoff, Professor of Electrical Engineering, Cornell University, before a conference of teachers in electrical engineering in Pittsburgh, Pa., July 25th, 1928. I. On Professional Development of Teachers in Advanced Branches of Electrical Engineering. II. On Electric Transients. In his first discourse, Professor Karapetoff in relation to electrical engineering deals with a condition of American education which is arousing marked concern among many of the leaders of scholarship of the country. In the course of a recent article on the mass-production of culture in the United States, Doctor Paul Shorey in this connection says: "It is a common indictment of American education that the average college graduate of ten years standing cannot be distinguished from the now-collegiate travelling man in the smoking car by his grammar, his vocabulary or the range of his information." He concludes: "In general and cultural education, my personal impression is that there has been a distinct decline since my own college days and early years of teaching. Many thoughtful teachers and writers seem to share the same opinion."

The discussion is limited to those teachers who guide seniors and graduate students, instructors in general and elementary branches being specifically excluded. Doctor Karapetoff specifies the theme of his first discourse in two divisions: (*a*) The total contribution to the substantial progress of electrical science and art by professors of electrical engineering, year after year, remains small in quantity and unimportant in quality. (*b*) Work done by graduate students in most colleges of engineering is small in quantity and inconsequential in quality and in results.

It is possible that powerful natural causes beyond anyone's control govern this situation. As such, may be mentioned: isolation from the rest of the world, privileged economic position; rapid commercial expansion; intensely practical ideas; material prosperity; ignorance of foreign tongues, unwillingness to sacrifice some of the standard of living and to exert a sustained mental effort in a single direction; interest in elementary teaching; general shiftlessness, with a pronounced tendency towards administrative and commercial jobs; side issues and social obligations; poor preparation in mathematics, physics, and methods of attack and analysis; over-use of the self suggestion about being underpaid and not having the necessary facilities; employment of high-grade experts by manufacturing and operating companies and a large amount of research done by them; etc.

The speaker's views of the proper functions of professors in advancing the electrical art are defined, his estimate of the superiority of German, and to some extent the British professors are pointed

out, and suggestions are made whereby professors of electrical engineering may hope to attain the leadership and vision such as exists in Germany and England.

The second address specifically deals with the teaching of electric transients under which term are included the phenomena and the theory of operation of electric circuits, apparatus, and machinery under variable conditions, especially during periods of starting and stopping, on initial short-circuit, on sudden open-circuit, rapid changes in load or voltage, arcover, etc. More and more are engineers being called upon to deal with transient phenomena. The speaker discusses why transient phenomena should be taught, what should be taught among transient phenomena, how they should be taught and where to find time in the curriculum necessary for such a course. The required preparation on the part of the student and the qualifications of teachers conclude the address.

L. E. P.

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